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APR 80 J C LAI, J M SIMMONS

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Numerical Solution of Steady and Periodically Pulsed Two-Dimensional Turbulent Free Jets

by

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and

J. M. Simmons

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NOMENCLATURE

Unless otherwise stated, the symbols used in the text have the following meanings.

a	constant defined in Eq. (15a)
b	constant defined in Eq. (15b)
c	constant defined in Eq. (11)
F	% peak-to-peak oscillation of centre-line velocity
f	stream function in (ζ,t,η) coordinates defined in Eq. (15b)
h	nozzle width
M	mean momentum flux in streamwise direction
P^*	mean pressure
p^*	instantaneous pressure
Q	mean mass flow at any streamwise station
Q_E	mean mass flow at nozzle exit
T	period of pulsation
t	non-dimensional time = $U_{ci}{}^*t^*/h$
U	non-dimensional mean x-component velocity = $U^*/U_{ci}{}^*$
$U_o(y)$	non-dimensional mean velocity profile at the nozzle exit
u	non-diemnsional instantaneous x-component velocity = $u^*/U_{ci}{}^*$
u'	non-dimensional x-component velocity fluctuation = $u^{*\prime}/U^*_{ci}$
V	non-dimensional mean y-component velocity = V^*/U^*_{ci}
v	non-dimensional instantaneous y-component velocity = v^*/U^*_{ci}

v'	non-dimensional y-component velocity fluctuation = $v^{*\prime}/U^*_{ci}$
x	non-dimensional streamwise distance - x^*/h
y	non-dimensional transverse distance - y^*/h
$y_{1/2}$	value of y at which $U = \frac{1}{2}U_c$

GREEK SYMBOLS

Δ	difference between two quantities
ε	amplitude of pulsation
ν	non-dimensional kinematic viscosity = $\nu^*/(U^*_{ci}h)$
ν_t	eddy-viscosity defined in Eq. (10)
ν_{eff}	Effective eddy viscosity defined in Eq. (13)
ψ	stream function in (ζ,t,y) coordinates defined in Eq. (15b)
ϕ	phase angle
Φ	function defined in Eq. (28)
η	transformed variable defined in Eq. (15b)
ω	non-dimensional angular frequency of pulsation = $\omega^*h/U_{ci}{}^*$
ζ	non-dimensional distance identical to x
ζ_0	constant defined in Eq. (15a)

SUPERSCRIPT

*	dimensional quantities

SUBSCRIPTS

c	centerline
i	initial
∞	jet edge
Overbar	time-average quantities

1.0 Introduction

The steady two-dimensional turbulent jet has long been the subject of many theoretical and experimental investigations because of its important role in many different types of engineering applications such as fluidic and combustion systems and because of its significance in providing fundamental understanding to the physics of flow mechanisms such as turbulence and vortex structures. Owing to its simplicity in configuration, the steady two-dimensional turbulent jet has been studied in detail experimentally and is regarded as one of the most well-documented flows in the literature where sufficient data are available for many practical engineering purposes. However, despite such extensive investigations, for example Heskestad (1965), Gutmark and Wygnanski (1976) and Everitt and Robins (1978), considerable scatter is found to exist between the results of various workers even in the mean flow parameters such as centre-line velocity decay rate and jet spreading rate. Such discrepancies have been casually attributed to different effects such as Reynolds number, aspect ratio, nozzle geometry, initial conditions, upstream turbulence intensities and the uncertainties involved in the hot wire results in regions where reversed flow may occur. Nevertheless, no unified agreement on the effects of such factors and other flow mechanisms on the flow development has been reached. A

comprehensive review and evaluation of the experiemntal data on steady turbulent jets was given by Harsha (1971) and Rodi (1975).

The analytical solution of the fully-developed steady turbulent jet was first sought by Tollmien (1926) followed by Goertler (1942) and Schlichting (1965). With the advent of computer technology and the rapid development of numerical techniques, the steady turbulent jets have commonly been used for turbulence modelling and as standard test cases of turbulence models. Many numerical predictions of steady turbulent jets using various turbulence models have been attempted such as Rodi and Spalding (1970), Launder et al (1972) and Chen and Nikitopoulos (1979) to calculate the flow properties and to complement the experimental results.

Although almost inevitably unsteadiness of varying degrees occurs in practice either desirably to achieve certain favorable characteristics or undersirably due to the fluctuations in the surrounding fluid, very few results appear to have been reported on unsteady turbulent jets both theoretically and experimentally. Only until recently, because of the growing realization of the fundamental and practical implications of an improved understanding of unsteady effects, the excitation of turbulent jets by acoustic (Fiedler and Korschelt (1979)) mechanical (Bremhorst and Harch (1979)) and fluidic (Piatt and Viets (1979)) means has received considerable attention. On the other hand,

closed form solutions of the unsteady jets which adequately describe the flow development can hardly be obtained despite the efforts of Pai (1965) and McCormack et al (1966). Numerical solution of unsteady laminar jets was obtained by Kent (1973). Although turbulence models have been developed to give sufficiently accurate predictions of a wide variety of steady flows, the applicability of such turbulence models to unsteady flows is uncertain.

The objectives of this study are to apply the transformation developed by Lai and Simmons (1978) to compute steady and unsteady turbulent jets, to add to the understanding of the steady and unsteady two-dimensional turbulent free jets, to investigate the validity of quasi-steady approximations and to evaluate the suitability of using turbulence models established for steady flows in unsteady flow calculations. The unsteady jet considered consists of an initially steady, two-dimensional, turbulent free jet with a sinusoidal mass flow variation superimposed on it at the nozzle exit. The steady-state oscillatory flow characteristics at any location downstream of the nozzle are obtained by solving the thin shear layer equations in two spatial (ζ,η) and one time (t) transformed coordinates.

2.0 Governing Equations

The Navier-Stokes equations for two-dimensional incompressible flow in tensor notations are given by (see, e.g., Hinze (1975))

$$\frac{\partial U_i^*}{\partial t^*} + u^*_j \frac{\partial U_i^*}{\partial x_j^*} = - \frac{1}{\rho} \frac{\partial p^*}{\partial x_i^*} + \nu^* \frac{\partial^2 U_i^*}{\partial x_j^* \, \partial x_j^*} \tag{1}$$

with the continuity equation being given by

$$\frac{\partial u_i^*}{\partial x_i^*} = 0 \tag{2}$$

By applying Reynolds decomposition

$$u_i^* = U_i^* + u_i^{*\prime} \text{ , etc.}$$

to equation (1) and taking time average with a time scale large compared with that of the turbulent motions but small compared with the periodicity of the flow, the time-averaged Reynolds equations can then be given by

$$\frac{\partial U_i^*}{\partial t^*} + U_j^* \frac{\partial U_i^*}{\partial x_j^*} = \frac{1}{\rho} \frac{\partial}{\partial x_j^*}\left(-P^* + \frac{\partial U_i^*}{\partial x_j^*} + \frac{\partial U_j^*}{\partial x_i^*} - \overline{\rho u_i^{*\prime} u_j^{*\prime}}\right) \tag{3}$$

Consider an unsteady, two-dimensional, constant property, turbulent free jet issuing into a stationary medium. The instantaneous configuration is shown schematically in Fig. 1.

Assuming the thin shear layer approximations as in Cebeci and Bradshaw (1977), the governing equations can be obtained from equations (2) and (3) to yield in rectangular coordinate system

$$\frac{\partial U^*}{\partial x^*} + \frac{\partial V^*}{\partial y^*} = 0 \tag{4a}$$

and

$$\frac{\partial U^*}{\partial t^*} + U^*\frac{\partial U^*}{\partial x^*} + V^*\frac{\partial U^*}{\partial y^*} = \frac{\partial}{\partial y^*}\left(\nu^*\frac{\partial U^*}{\partial y^*} - \overline{u^{*\prime}v^{*\prime}}\right) \tag{4b}$$

The boundary conditions on the centre-line and at the edge of the jet can be expressed respectively by

$$t^* \geq 0 \quad \begin{cases} y^* = 0 & \dfrac{\partial U^*}{\partial y^*} = V^* = 0 & \text{(5a)} \\ y^* = y_\infty & U^* = 0 & \text{(5b)} \end{cases}$$

The time-varying boundary condition at the nozzle exit is given by

$$t^* \geq 0 \qquad x^* = 0 \qquad U^* = U_o^*(y^*)(1 + \varepsilon\sin\omega^* t^*) \tag{6a}$$

where $U_o^*(y^*)$ is the mean velocity profile at the nozzle exit, ε is the amplitude of pulsation and ω^* is the angular frequency of pulsation.

For all locations downstream of the nozzle, the initial conditions are given by

$$t^* = 0 \qquad x^* \geq 0 \qquad U^* = U_i^*(x^*, y^*) \tag{6b}$$

where $U_i^*(x^*,y^*)$ is the steady state solution of equation (4) with the boundary condition at the nozzle exit given by equation (6a) with $t^* = 0$.

2.1 Governing Equations in Non-Dimensional Form:

In order to enable the solution obtained to be valid for a family of velocity profiles which have the same normalized shape at the nozzle exit, the following non-dimensional variables are used: -

$$U(x,y,t) = \frac{U^*(x^*,y^*,t^*)}{U_{ci}^*}, \quad V(x,y,t) = \frac{V^*(x^*,y^*,t^*)}{U_{ci}^*}$$

$$x = \frac{x^*}{h} \qquad y = \frac{y^*}{h} \qquad t = \frac{U_{ci}^*}{h}t^* \qquad \omega = \frac{h}{U_{ci}^*}\omega^*$$

$$\nu = \frac{\nu^*}{U_{ci}^* h} \qquad \overline{u'v'} = \frac{\overline{u^{*\prime}v^{*\prime}}}{U_{ci}^{*2}}$$

where U_{ci}^* is the centre-line velocity at the nozzle exit at $t^* = 0$.

Equation (4) can now be written as

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{7a}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(\nu\frac{\partial U}{\partial y} - \overline{u'v'}\right) \tag{7b}$$

subject to the following boundary and initial conditions

$$t \geq 0 \quad \begin{cases} y = 0 & \dfrac{\partial U}{\partial y} = V = 0 & (8a) \\ y = \infty & U = 0 & (8b) \end{cases}$$

$$t \geq 0 \qquad x = 0 \qquad U = U_o(y)(1 + \varepsilon\sin\omega t) \tag{9a}$$

$$t = 0 \qquad x \geq 0 \qquad U = U_i\ (x,y) \tag{9b}$$

2.2 Turbulence Modelling

The Navier-Stokes equations constitute a complete set of equations of motion which can in theory be solved to yield a solution for any laminar or turbulent flow field. However, turbulence comprises a wide range of length scales bounded from above by the dimensions of the flow field and bounded from below by the diffusive action of molecular viscosity. The resolution scale of the smaller eddies which are responsible for the decay of turbulence is too small that it precludes the use of any existing computer. Futhermore, very often only the time-averaged properties are of interest in engineering applications even if the flow is time dependent. Hence, the Navier-Stokes equations are time-averaged first before being solved. This avoids not only the difficulty in representing all the characteristic turbulence scales but also unnecessary computations of transients if only the time-averaged properties are required. The process of time-averaging such as the Reynolds time-averaging described in section 2.0 causes the loss of certain information contained in the original equations and results in more unknowns than the governing equations through the introduction of statistical correlations of fluctuating velocities such as the $\overline{u'v'}$ term in equation (7b) which are known as apparent Reynolds stresses and are responsible for the actual momentum

transfer. Attempts to derive additional equations for those Reynolds stresses will only result in additional unknowns. Thus the time-averaging process presents a closure problem which is to reduce the number of unkonwns to equal the number of governing equations. In order to achieve this, the additional unknown quantities must be modelled or approximated in terms of known quantities through a set of equations which, when solved with the mean-flow equations, simulate the actual flow situation. This process is generally termed "turbulence modelling."

Turbulence models can broadly be classified into "first-order" models in which the mean flow equations are solved without additional partial differential equations for the velocity fluctuation terms and "higher-order" models in which transport equations for higher-order velocity correlations are solved with the mean flow equations. Since in the "first-order" models additional partial differential equations are not solved for the turbulence quantities which are expressed in terms of known quantities through certain algebraic formulation, they are known as zero-equation models and "higher-order" models which involve at least one partial differential equation for the turbulence quantities can thus be termed one-equation or two-equation models, etc. A description of the various types of turbulence models can be found in Launder and Spalding (1972) and a comprehensive review of the state-of-art is given by Reynolds and Cebeci (1978).

Zero-equation models which are based on empirical correlations of the extensive available experimental data and mostly on the eddy-viscosity and mixing-length concepts such as the Cebeci-Smith eddy viscosity model (1974) have been widely used and proved to be very successful in obtaining very accurate predictions of non-separating flows. For more complex flows which involve recirculation and separation, the more refined two-equation k-ε model described by Launder and Spalding (1976) is the most well-developed among all other "high-order" models. It has been recognized that although "higher-order" models contain more information and hence simulate the flow situation more realistically than the "first-order" models, they are more difficult to solve and require generally an order-of magnitude more computing storage and time while in non-separating and simple flow situations, the first-order models can yield predictions to the same degree of accuracy as the "higher-order" models. It is in the light of this philosophy that in this study, a constant-eddy viscosity model due to Prandtl (1942) is used.

By employing the eddy-viscosity concept of Boussinesq, the fluctuating-velocity correlation term $\overline{u'v'}$ in equation (7b) is related to the mean velocity gradient through the eddy viscosity ν_t as follows:

$$\overline{u'v'} = -\nu_t \frac{\partial U}{\partial y} \qquad (10)$$

The eddy viscosity ν_t can then be obtained from the constant eddy-viscosity model as

$$\nu_t(t) = c y_{1/2} U_c(t) \qquad (11)$$

where c is a constant, and $y_{1/2}$ is the jet half-width.

In this model, two constants c_1 and c_2 are used for the free jet as it emerges from developing to fully developed form. Hence c is given by

$$c = \begin{cases} c_1 & \text{if } \bar{U}_c \geqslant 0.95 \\ c_2 & \text{if } \bar{U}_c < 0.95 \end{cases} \qquad (12)$$

By defining an effective eddy viscosity ν_{eff} such that

$$\nu_{eff} = \nu_t + \nu \qquad (13)$$

and using equation (10),equation (7b) can then be written as

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}(\nu_{eff}\frac{\partial U}{\partial y}) \qquad (14)$$

Hence the flow development of a periodically pulsed turbulent free jet is governed by equations (7a) and (14) subject to the boundary and initial conditions of equations (8) and (9) with the constant eddy viscosity model of (11)

2.3 Governing Equation in Transformed Variable Form

It has been shown by Lai and Simmons (1978) that tranformations based on the similarity solutions are successful in reducing substantially the rate of spread of an unsteady laminar jet, hence enabling the use of variable grid sizes over predetermined regions. In this study, similar transformations are employed. A dimensionless transverse distance η and a dimensionless stream function ψ are defined by

$$\eta = ay/(\zeta + \zeta_o) \tag{15a}$$

$$\psi(\zeta,y,t) = b(\zeta + \zeta_o)^{1/2} f(\zeta,\eta,t) \tag{15b}$$

where a, b and ζ_o are arbitrary constants which can be varied to facilitate computation and x is renamed as ζ .

The function f automatically satisfies the continuity equation (7a) and equation (14) can be re-written in the transformed coordinates as

$$(\zeta + \zeta_o)^{-1/2}[\nu_{eff} f'']' + (f')^2 + ff'' = 2(\zeta + \zeta_o)[f'\frac{\partial f'}{\partial \zeta} - f''\frac{\partial f}{\partial \zeta} + \frac{(\zeta + \zeta_o)^{1/2}}{ab} \frac{\partial f'}{\partial t}] \tag{16}$$

where prime denotes differentiation with respect to η ,

$$f' = (\zeta + \zeta_o)^{1/2}U/ab \tag{17}$$

and a and b are chosen for convenience such that $a/b = \frac{1}{2}$.

The boundary conditions in equation (8) become

$$t > 0 \begin{cases} \eta = 0 \qquad f'' = 0 \qquad f + 2(\zeta + \zeta_o)\frac{\partial f}{\partial \zeta} = 0 & (18a) \\ \eta = \eta_\infty \qquad f' = 0 & (18b) \end{cases}$$

The initial conditions at $\zeta = 0$ in the (η , t) plane is obtained by writing equation (9a) with equation (17) as

$$f' = f_o'(\eta)(1 + \varepsilon\sin\omega t) \tag{19a}$$

The initial conditions at $t = 0$ in the (ζ,η) plane are generated by the solutions $U_i(\zeta,\eta)$ of the following steady jet equation obtained from equation (16)

$$(\zeta + \zeta_o)^{-\frac{1}{2}} \quad [\nu_{eff}f'']' + (f')^2 + ff'' = 2(\zeta + \zeta_o)[f'\frac{\partial f'}{\partial \zeta} - f''\frac{\partial f}{\partial \zeta}] \tag{19b}$$

The effective eddy-viscosity can be obtained from equation (13) using equations (11) and (15) to give

$$\nu_{eff}(t) = cb(\zeta + \zeta_o)^{1/2} \; \eta_{1/2}f_c'(t) + \nu \tag{20}$$

where c is subject to the conditions specified in equation (12).

3.0 Method of Solution

Since the initial conditions in the (ζ,η) plane are generated by solving the steady jet equation (19b) subject to the boundary conditions in equation (18), the flow development of the steady jet is first studied. For computer programme development purpose, two separate computer programs were written for the steady and unsteady jet respectively with the solutions of the steady jet stored on disk for use in the unsteady jet program. However, if necessary, the steady jet program can easily be incorporated into the unsteady one. Both equations (16) and (19b) are parabolic and can be solved by a marching procedure. The finite difference scheme employed is the Box Method developed by Keller (1970), and described in detail by Cebeci and Bradshaw (1977). The scheme has been applied successfully to a wide range of boundary-layer type of flows including both time-dependent and separating flows by Cebeci and his co-workers (e.g., see Bradshaw et al (1980)).

3.1 Finite Difference Form of the Governing Equations

Equation (16) is rewritten as a system of first order parital differential equations -

$$f' = g \tag{21a}$$

$$g' = q \tag{21b}$$

$$(\zeta + \zeta_o)^{-1/2}[\nu_{eff}\, q]' + g^2 + fq = 2(\zeta + \zeta_0)[g\frac{\partial g}{\partial \zeta} - q\frac{\zeta f}{\partial \zeta} + (\frac{\zeta + \zeta_o}{ab})^{1/2} \frac{\partial g}{\partial t}] \tag{21c}$$

Consider the net cube shown in Fig. 2 and denote the grid points by

$$\zeta_0 = 0 \quad \zeta_{i+1} = \zeta_i + r_i \quad i = 1,2\ldots, I$$

$$t_o = 0 \quad t_n = t_{n-1} + k_n \quad n = 1,2,\ldots, N$$

$$\eta_0 = 0 \quad \eta_j = \eta_{j-1} + h_j \quad j = 1,2,\ldots, J$$

$$\eta_J = \eta_\infty$$

where i,n,j are just sequence numbers and the variable net spacings r_i, k_n and h_j are completely arbitrary.

In the study of jets, a coarse grid spacing h_j can be used in the vicinity of the centre line and the edge of the jet as the transverse (η) gradients of f are small in those regions. The quantities (f,g,q) at points (ζ_i, t_n, η_j) are approximated by the grid functions ($f_j^{i,n}$, $g_j^{i,n}$, $q_j^{i,n}$). Hence by using central differencing and averaging about the mid-point (ζ_i, t_n, $\eta_{j-½}$), Eqs. (21a) and (21b) can be written in the following finite difference forms

$$(f_j^{i,n} - f_{j-1}^{i,n})/h_j = g_{j-\frac{1}{2}}^{i,n} \tag{22a}$$

$$(g_j^{i,n} - g_{j-1}^{i,n})/h_j = q_{j-\frac{1}{2}}^{i,n} \tag{22b}$$

where, for example, the shorthand notation $g_{j-\frac{1}{2}}^{i,n}$ has been used for $\frac{1}{2}(g_j^{i,n} + g_{j-1}^{i,n})$

Shorthand notations are introduced, for example,

$$\bar{\bar{f}}_i = \tfrac{1}{2}(f_j^{i,n} + f_j^{i,n-1} + f_{j-1}^{i,n} + f_{j-1}^{i,n-1}) = \tfrac{1}{2}(f_{j-\frac{1}{2}}^{i,n} + f_{j-\frac{1}{2}}^{i,n-1})$$

$$\bar{f}_j = \tfrac{1}{4}(f_j^{i,n} + f_j^{i-1,n} + f_j^{i,n-1} + f_j^{i-1,n-1}) = \tfrac{1}{4}(f_j^{i,n} + f_j^{234})$$

$$\bar{f}_n = \tfrac{1}{4}(f_j^{i,n} + f_j^{i-1,n} + f_{j-1}^{i,n} + f_{j-1}^{i-1,n}) = \tfrac{1}{2}(f_{j-\frac{1}{2}}^{i,n} + f_{j-\frac{1}{2}}^{i-1,n})$$

$$f_j^{234} = f_j^{i,n-1} + f_j^{i-1,n} + f_j^{i-1,n-1}$$

Eq. (21c) can be approximated using central differencing at $(\zeta_{i-\frac{1}{2}}, t_{n-\frac{1}{2}}, \eta_{j-\frac{1}{2}})$ by

$$(Bq)_j - (Bq)_{j-1} + h_j\{(g^2)_{j-\frac{1}{2}} + (fq)_{j-\frac{1}{2}} - \alpha[g_{j-\frac{1}{2}}(g_{j-\frac{1}{2}} + g_{j-\frac{1}{2}}^{i,n-1} + g_{j-\frac{1}{2}}^{234} - 2\bar{g}_{i-1}) - q_{j-\frac{1}{2}}(f_{j-\frac{1}{2}} + f_{j-\frac{1}{2}}^{i,n-1} - 2\bar{f}_{i-1}) - f_{j-\frac{1}{2}}q_{j-\frac{1}{2}}^{234}] - \alpha_i g_{j-\frac{1}{2}}\} = T_{j-\frac{1}{2}}^{i-1,n-1} \tag{22c}$$

where $B = (\zeta + \zeta_0)^{-\frac{1}{2}}\nu_{eff}$

$$\alpha = \frac{m_1^{i-\frac{1}{2}}}{2r_i}$$

$$m_1 = 2(\zeta + \zeta_0)$$

$$\alpha_1 = \frac{4[(\zeta + \zeta_0)^{3/2}]^{i-\frac{1}{2}}}{ab\ k_n}$$

$$T_{j-\frac{1}{2}}^{i-1,n-1} = (Bq)_{j-1}^{234} - (Bq)_j^{234} + h_j \{-(g^2)_{j-\frac{1}{2}}^{234} - (fq)_{j-\frac{1}{2}}^{234} + a[\beta_2\ g_{j-\frac{1}{2}}^{234} - \beta_1 q_{j-\frac{1}{2}}^{234}] + \alpha_1\beta_3\}$$

$$\beta = g_{j-\frac{1}{2}}^{i,n-1} + g_{j-\frac{1}{2}}^{234} - 2\ \bar{g}_{i-1}$$

$$\beta_1 = f_{j-\frac{1}{2}}^{i,n-1} + 2\ \bar{f}_{i-1}$$

$$\beta_2 = g_{j-\frac{1}{2}}^{i,n-1} - 2\ \bar{g}_{i-1}$$

$$\beta_3 = g_{j-\frac{1}{2}}^{i-1,n} - 2\ \bar{g}_{n-1}$$

In the above equation (22c), the superscripts i,n have been dropped for simplicity.

It has been pointed out by Keller (1978) that in approximating non-linear terms such as $(fq)_{j-\frac{1}{2}}$ in equation (22c) with the Box scheme, there exists several choices which should not have serious effect on accuracy or stability as long as the proper centering is maintained. However, in jet calculations, it has been found in this study that the approximation of terms such as (fq) in the form of $(f)_{j-\frac{1}{2}}(q)_{j-\frac{1}{2}}$

results in better numerical stability at the jet edge. The boundary conditions in Eq. (18) become

$$q_0^{i,n} = 0 \tag{23a}$$

$$f_0^{i,n} = (1 + \frac{2m_1}{r_i})^{-1}(\frac{2m_1}{r_i} - 1)f_0^{i-1,n} \tag{23b}$$

$$g_J^{i,n} = 0 \tag{23c}$$

The initial conditions at $\zeta = 0$ in the (η, t) plane in Eq (19a) are given by

$$(f')_j^n = (f'_o)_j^n[1 + \varepsilon \sin \omega(n\Delta t)] \tag{24}$$

where Δt is the temporal grid size.

The initial conditions at $t = 0$ in the $(\zeta.\eta)$ plane correspond to the steady jet solutions of equation (19b). The finite difference equations can be obtained similar to the above derivation for the unsteady jet except terms in equation (22) at $t = t_{n-1}$ are equal to those at $t = t_n$ and $\alpha_1 = 0$.

3.2. Time Varying Velocity Profile at the Nozzle Exit

Because of symmetry, only half of the jet needs to be computed. The time-varying velocity profile considered at the nozzle exit is the commonly assumed "top-hat" profile which is given by

$$U(y,t) = \begin{cases} 1 + \varepsilon \sin \omega t & \text{for } 0 \leqslant y \leqslant 1/2 \\ 0 & \text{for } 1/2 < y \end{cases} \tag{25}$$

In terms of the transformed coordinates, equation (25) can be rewritten using equation (17) as

$$f' = \begin{cases} \dfrac{(\zeta + \zeta_o)^{1/2}}{ab} (1 + \varepsilon \sin \omega t) & \text{for } 0 \leq \eta \leq (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \leq y \end{cases}$$

$$\text{where } (\eta_\infty)_i = 0.5\ a/\zeta_o \tag{26}$$

Since a discontinuity exists in f' at $\eta = (\eta_\infty)_i$ and will cause computational problems, the top-hat profile has to be approximated by

$$f' = \frac{(\zeta + \zeta_o)^{1/2}}{ab} \Phi_1 (\eta)(1 + \varepsilon \sin \omega t) \tag{27}$$

where

$$\Phi_1(\eta) = \begin{cases} 1 & \text{for } 0 \leq \eta \leq \eta_a \\ (1 - \eta_N{}^2)(1 + 2\eta_N) & \text{for } \eta_a \leq \eta \leq (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \leq \eta \end{cases} \tag{28}$$

$$\eta_N = \frac{\eta - \eta_a}{(\eta_\infty)_i - \eta_a}$$

and η_a is some point in the interval $[0, (\eta_\infty)_i]$.

The function $(1 - \eta_N^2)(1 + 2\eta_N)$ was obtained by matching the velocity and velocity gradient profiles at $\eta = (\eta_\infty)_i$ and $\eta = \eta_a$ through the following boundary conditions to ensure continuity:

$$\eta_N = 0 \qquad \Phi_1 = 1 \qquad \Phi_1' = 0 \tag{29a}$$

$$\eta_N = 1 \qquad \Phi_1 = 0 \qquad \Phi_1' = 0 \tag{29b}$$

f and f'' can be obtained by respectively integrating and differentiating equation (27) with respect to η to yield

$$f = \frac{(\zeta + \zeta_0)^{1/2}}{ab} \quad \Phi_o(\eta)(1 + \varepsilon \sin \omega t) \tag{30}$$

and
$$f'' = \frac{(\zeta + \zeta_0)^{1/2}}{ab} \quad \Phi_2(\eta)(1 + \varepsilon \sin \omega t) \tag{31}$$

$$\Phi_o(\eta) = \begin{cases} 1 & \text{for } 0 \le \eta \le \eta_a \\ [(\eta_\infty)_i - \eta_a]\eta_N(1 - \eta_N^2 + 0.5\eta_N^3) & \text{for } \eta_a \le \eta \le (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \le \eta \end{cases} \tag{32}$$

and
$$\Phi_2(\eta) = \begin{cases} 1 & \text{for } 0 < \eta < \eta_a \\ -6\eta_N (1 - \eta_N)/[(\eta_\infty)_i - \eta_a] & \text{for } \eta_a \le \eta \le (\eta_\infty)_i \\ 0 & \text{for } (\eta_\infty)_i \le \eta \end{cases} \tag{33}$$

The value of η_a can be determined to match the experimental top-hat profile as close as possible and in this study it is chosen to be the point such that the velocity profile is uniform over 97.5% of the nozzle.

Adopting similar procedure given in Lai and Simmons (1978), a and ζ_o are given the values 0.3 and 3.75 respectively such that $(\eta_\infty)_i = 0.04$ and $\eta_a = 0.039$.

3.3 Solution of the Finite Difference Equation

With $(f_j^{n-1}, g_j^{n-1}, q_j^{n-1})$ known from the solution of Eq (19b) and $(f_j^{i-1}, g_j^{i-1}, q_j^{i-1})$ specified by Eqs. (27), (30) and (31), Eq.(22) for $1 \leq j \leq J$ and Eq. (23) yield an implicit non-linear algebraic system of 3J + 3 equations. This system is linearized by Newton's Method by introducing the perturbed quantities $(\delta f, \delta g, \delta q)$ to yield after considerable algebra

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}(\delta g_j + \delta g_{i-1}) = (r_i)_j \tag{34a}$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}(\delta q_j + \delta q_{j-1}) = (r_3)_{j-1} \tag{34b}$$

$$(s_1)_j\ \delta q_j + (s_2)_j\ \delta q_{j-1} + (s_3)_j\ \delta f_j + (s_4)_j \delta f_{j-1}$$

$$+ (s_5)_j \delta g_j + (s_6)_j \delta g_{j-1} = (r_2)_j \tag{34c}$$

where

$$(s_1)_j = B_j + \frac{h_j}{2}(f_j + \alpha f_{j-\frac{1}{2}} + \alpha\beta_1)$$

$$(s_2)_j = B_{j-1} + \frac{h_j}{2}(f_{j-1} + \alpha f_{j-\frac{1}{2}} + \alpha\beta_1)$$

$$(s_3)_j = \frac{h_j}{2}(q_j + \alpha q_{j-\frac{1}{2}} + \alpha q_{j-\frac{1}{2}}^{234})$$

$$(s_4)_j = \frac{h_j}{2}(q_{j-1} + \alpha q_{j-\frac{1}{2}}^{234})$$

$$(s_5)_j = h_j(g_j - \alpha g_{j-\frac{1}{2}} - \frac{\alpha\beta}{2} - \frac{\alpha_1}{2})$$

$$(s_6)_j = h_j(g_{j-1} - \alpha g_{j-\frac{1}{2}} - \frac{\alpha\beta}{2} - \frac{\alpha_1}{2})$$

$$(r_1)_j = f_{j-1} - f_j + h_j\ g_{j-\frac{1}{2}}$$

$$(r_3)_{j-1} = g_{j-1} - g_j + h_j\, q_{j-\frac{1}{2}}$$

$$\begin{aligned}(r_2)_j &= T^{i-1,n-1}_{j-\frac{1}{2}} + (Bq)_{j-1} - (Bq)_j - h_j\, \{g^2_{j-\frac{1}{2}}\, (fq)_{j-\frac{1}{2}} \\ &\quad - \alpha[\tfrac{1}{2}(g^2_{j-\frac{1}{2}} + g_j\, g_{j-1}) + \beta\, g_{j-\frac{1}{2}} - f_{j-\frac{1}{2}} g_{j-\frac{1}{2}} \\ &\quad - \beta_1\, q_{j-\frac{1}{2}} - q^{234}_{j-\frac{1}{2}}\, f_{j-\frac{1}{2}}] - \alpha_1\, g_{j-\frac{1}{2}}\}\end{aligned}$$

The boundary conditions are given by

$$\begin{aligned}\delta f_0 &= 0 \\ \delta q_0 &= 0 \\ \delta g_j &= 0\end{aligned} \tag{35}$$

The above derivation is similar to that outlined in Appendix C in Lai & Simmons (1978). The linear system Eq. (3.10) is then solved very effectively by the Block Elimination Method discussed by Cebeci and Bradshaw (1977).

The linearized form of the steady jet equation can similarly be obtained and solved with quantities at $t = t_{n-1}$ equal to those at $t = t_n$ and $\alpha_1 = 0$.

3.4 Convergence Criterion

As the governing equations for both the steady and unsteady jet are parabolic, they are solved by marching along the t-direction in the case of the unsteady jet and along the ζ-direction in the case of the steady jet. At a given streamwise (ζ) station, the linearized system Eq. (34) is solved by iterating at each t-station until some convergence criterion is satisfied. Iterations are terminated at each t-station if

$$|f'^{(i+1)} - f'^{(i)}| < \varepsilon_1 \text{ , at } \eta = \eta_{con} \qquad (36)$$

where the value of ε_1 is prescribed;

$f'^{(i)}$ and $f'^{(i+1)}$ are the i th and (i+1)th iterates of f' respectively; and η_{con} is some point where the convergence criterion is applied. In practice, it is adequate to set ε_1 to be 10^{-3} .

3.5 Criterion for the Spreading of Jet

Since the mass flow varies sinusoidally with time, and because of the initially steep velocity gradient, the jet width will be changing with time at a given streamwise (ζ) station. A criterion must therefore be set to determine η_∞ at each t-station, noting that $\eta_\infty^{(i)} \geq \eta_\infty^{(i-1)}$. Here $\eta_\infty{}^{(i)}$ and $\eta_\infty^{(i-1)}$ are the jet width for the i th and (i-1)th iterations. The edge of the jet is defined by the following two conditions -

$$|f'_{J-1}| \leq \varepsilon_2 \tag{37a}$$

$$|f''_j \quad | \leq \varepsilon_3 \tag{37b}$$

where the values of ε_2 and ε_3 are prescribed and J denotes the point at the jet boundary. Experience indicates that it is sufficient to choose

$$\varepsilon_2 = 10^{-2} \quad \text{and} \quad \varepsilon_3 = 10^{-1}$$

If the criteria set out in Eq. (37) are satisfied then $\eta_\infty^{(i+1)} = \eta_\infty^{(i)}$. Otherwise n_g points have to be added so that $J_{new} = J_{old} + n_g$ and the values of $(f_j{}^{i,n}\ g_j{}^{i,n},\ q_j^{i,n})$ for the new η_j points are obtained as follows -

$$f_j^{i,n} = (\eta_j - \eta_\infty)\, g_J^{i,n} + f_J^{i,n} \tag{38a}$$

$$g_j^{i,n} = g_J^{i,n} \tag{38b}$$

$$q_j^{i,n} = 0 \tag{38c}$$

$$B_j^{i,n} = B_J^{i,n} \tag{38d}$$

The same procedure is also applied to $f_j^{i,n-1}$, $g_j^{i,n-1}$, $q_j^{i,n-1}$, $B_J^{i,n-1}$, $f_j^{i-1,n}$, $g_j^{i-1,n}$, $q_j^{i-1,n}$, $B_j^{i,n-1}$, $f_j^{i-1,n-1}$, $q_j^{i-1,n-1}$, $B_j^{i-1,n-1}$

3.6 Criterion for the Attainment of Steady State Solution

As the mass flow at the nozzle varies sinusoidally with time, the steady state solution at any streamwise (ζ) station downstream must vary periodically and can be expressed in the form

$$f'(\zeta,t,\eta) = f'_0(\zeta,t,\eta) + \lim_{L\to\infty} \sum_{\ell=1}^{L} \varepsilon_\ell \; f_\ell(\zeta,\eta) \sin(\ell\omega t + \phi_\ell) \quad (39)$$

In general, for a periodically pulsed flow, the steady state solution at a given streamwise (ζ_i) station, is said to be reached if

$$f'(\zeta,t + nT,\eta) = f'(\zeta,t,\eta) \quad \text{for} \quad n = 1.2,...$$

where T is the period of oscillation.

This entails velocity profiles to agree over a few cycles and thus requires unnecessary and uneconomic computations extending for a few periods in order to ascertain that steady state has been attained. However, if all the three parameters, f, f' and f" are considered, it is adequate to regard that the steady state solution has been reached if at some point η_s, the following criteria are satisfied -

$$\begin{aligned}
&|[f(\zeta,t_n + T,\eta_s) - f(\zeta,t_n,\eta_s)]/(f(\zeta,t_n,\eta_s)| \leq \varepsilon_4 \\
&|[f'(\zeta,t_n + T,\eta_s) - f'(\zeta,t_n,\eta_s)]/f'(\zeta,t_n,\eta_s)| \leq \varepsilon \qquad n = 1,2 \quad (40) \\
&|[f''(\zeta,t_n + T,\eta_s) - f''(\zeta,t_n,\eta_s)]/f''(\zeta,t_n,\eta_s)| \leq \varepsilon_6
\end{aligned}$$

where the values of ε_4, ε_5 and ε_6 are prescribed and $t_2 - t_1 = \Delta t$.

The sensitivity of the steady-state test varies with the point η_s to which it is applied. In practice, the point η_s should be so chosen such that reasonable sensitivity is achieved without excessive computations.

Since f' and f" are the first and second order derivatives of f , discrepancies between results separated by one period T are more pronounced in f" . Hence, in general ε_4, ε_5 and ε_6 must be so chosen that $\varepsilon_4 \lesssim \varepsilon_5 \lesssim \varepsilon_6$. Otherwise, if $\varepsilon_6 \gtrsim \varepsilon_5 \gtrsim \varepsilon_4$ and if ε_6 is very small, the criteria in Eq. (40) might never be satisfied although the steady state solution has long been attained within practical limits.

As a further check of the attainment of the steady-state condition, the following condition is imposed on f' at the centre-line -

$$|[f'(\zeta,t_n + T,0) - f'(\zeta,t_n,0)]/f'(\zeta,t_n,0)| \lesssim \varepsilon_7 \qquad (41)$$

for $n = 1,2$

where the value of ε_7 is prescribed.

It has been chosen that $\varepsilon_4 = \varepsilon_5 = \varepsilon_6 = \varepsilon_7 = 10^{-2}$

3.7 Mean Momentum Flux, Phase Angle and Peak-to-Peak Oscillation

3.7.1 Mean Momentum Flux

The instantaneous momentum at a specified streamwise (ζ_i) station is given by

$$M_i = 2hU_{ci}^{*2} \int_0^{\eta_\infty} [f'(\zeta, t_i, \eta)]^2 \, d\eta \tag{42}$$

M_i is evaluated by Simpson's rule for unequally spaced points derived in Appendix D in Lai & Simmons (1978).

The mean momentum flux in the streamwise direction is defined by

$$M = \frac{1}{T} \int_{t_0}^{t_0 + T} M_i \, dt \tag{43}$$

and the integral is approximated by

$$M = \frac{1}{N} \sum_{i=1}^{N} M_i \tag{44}$$

The quantity M can be served as an additional check on the overall results by testing for its constancy with ζ.

3.7.2. Phase Angle

The phase angle between the fundamental component of the centre-line velocity at any downstream station and that at the nozzle can readily be obtained by cross-correlating the steady-state instantaneous centre-line velocity at that station with a reference sine and cosine signal respectively.

Consider

$$S_1 = \sin \omega t \tag{45}$$

$$S_2 = \cos \omega t \tag{46}$$

From Eq. (39),

$$f_c' \equiv f'(\zeta,t,0) = f_0'(\zeta,t,0) + \lim_{L\to\infty} \sum_{\ell=1}^{L} \varepsilon^{\ell} \; f_{\ell}(\zeta,0) \; \sin(\ell\omega \; + \phi_{\ell}) \tag{47}$$

Multiplying Eq. (45) with Eq. (47) and taking time average yields

$$\overline{S_1 \; f_c'} = A \cos \phi_1 \tag{48}$$

where A is a constant.

Multiplying Eq. (46) with Eq. (47) and taking time average gives

$$\overline{S_2 f_c'} = A \sin \phi_1 \tag{49}$$

From Eqs. (48) and (49),

$$\phi_1 = \tan^{-1} \frac{\overline{S_2 f_c'}}{\overline{S_1 f_c'}} \tag{50}$$

3.7.3. Peak-to-Peak Oscillation

The percentage peak to peak variation of the centre-line velocity at any downstream station is defined by

$$F = \frac{(f_c')_{max} - (f_c')_{min}}{\overline{f}_c'} \times 100\% \tag{51}$$

The quantities $(f_c')_{max}$ and $(f_c')_{min}$ are obtained by quardratic interpolation of the form

$$f_c' = a_0\, t^2 + a_1\, t + a_2 \tag{52}$$

through three points $(f_c'\ (t_1),\ t_1)$, $(f_c'\ (t_2),\ t_2)$ and $(f_c'\ (t_3),\ t_3)$

where for $(f_c')_{max}$, $\quad f_c'(t_1) \lesssim f_c'\ (t_2)$

$$f_c'(t_3) \lesssim f_c'\ (t_2)$$

and for $(f_c')_{min}$ $\quad f_c'(t_1) \gtrsim f_c'\ (t_2)$

$$f_c'(t_3) \gtrsim f_c'\ (t_2)$$

Eq. (52) can be solved for a_o, a_1 and a_2 by substituting $f_c'(t_1)$, $f_c'(t_2)$ and $f_c'(t_3)$. With a_o, a_1 and a_2 known, $(f_c')_{max}$ or $(f_c')_{min}$ can be obtained from Eq. (52) with t given by $-a_1/2a_o$.

4.0 Results and Discussions

The structure and listing of the computer programs for both the steady and unsteady jet are described in Appendix A. Calculations were performed on the University of Queensland PDP1055 and the U.S. Naval Postgraduate School IBM 360/67 computers. The computer program for the steady jet occupies a core memory of 16K words and that of the unsteady jet occupies a core memory of 67K words. It must be pointed out here that the unsteady jet program can actually be reduced to about 25K words of core memory since at any one time instant, only calculations involving two time levels are required in core whereas all the quantities at other time levels can be stored on disk and retrieved when required. However, in this case, the computer core storage is not a problem with the computing facility available whereas the computing time is important and as such a trade-off is made such that quantities at all time levels are retained in core in order to save computing time of writing to and reading from disk. Moreover, although a constant eddy viscosity model was used, the programs were written to accept variable eddy viscosity. A very straightforward modification of the program by incorporating a routine to read from and write to a disk can reduce its size to enable it to be run on any mini-computer with available core memory of about 25K words.

4.1 Steady Jet

Owing to the nature of the initially top-hat velocity profile, the initial velocity gradient is very steep and the velocity profile changes vary appreciably over a very short streamwise distance. Since the solution of the finite difference equations is obtained through linearization by Newton's method as described in section 3.3, an initial guess at any streamwise station must be close to the solution. Consequently, a fine grid has also to be used initially but as the computation proceeds and as the velocity profiles start to appear in similar form, a coarser grid can be used by dropping every other point. The initial grid used in the transverse (η) direction is specified as follows:

$$\Delta\eta = \begin{array}{lr} 0.005 & 0 \leq \eta \leq 0.01 \\ 0.002 & 0.01 \leq \eta \leq 0.036 \\ 0.0002 & 0.036 \leq \eta \leq 0.039 \\ 0.0001 & 0.039 \leq \eta \leq 0.04 \\ 0.00025 & 0.04 \leq \eta \leq 0.045 \\ 0.001 & 0.045 \leq \eta \leq 0.06 \\ 0.002 & 0.06 \leq \eta \leq 0.02 \\ 0.01 & \eta \leq 0.12 \end{array} \tag{53}$$

$$(\eta_\infty)_i = 0.04$$

The grid-sizes in the streamwise (ζ) direction are specified as follows:

$$
\Delta\zeta = \begin{cases} 0.0005 & 0 \le \zeta \le 0.007 \\ 0.005 & 0.007 \le \zeta \le 0.032 \\ 0.025 & 0.032 \le \zeta \le 0.132 \\ 0.1 & 0.132 \le \zeta \le 1.032 \\ 1 & 1.032 \le \zeta \le 20.032 \\ 2 & 20.032 \le \zeta \le 40.032 \\ 5 & 40.032 \le \zeta \end{cases} \tag{54}
$$

The constants c_1 and c_2 in equation (12) in the constant eddy viscosity model are varied to match the experimental data. The sensitivity of the solutions to various values of c_1 and c_2 was tested. No significant difference was found when c_1 was varied from 0.009 to 0.012 and c_2 from 0.032 to 0.037. The final values of $c_1 = 0.009$ and $c_2 = 0.034$ were chosen.

As shown in Fig. 3, the non-dimensional self-preserved velocity profiles of various workers agree very well with each other and with the Goertler solution except near the jet edge where conventional hot-wire measurements are dubious. However, the results of Heskestad (1965) follow very closely the Goertler solution and those of Robins (1971) which have been recommended by Rodi (1975) as reliable agree very well with the results of Heskestad (1965). The computed development of the non-dimensional velocity profiles of the steady jet with y normalized with respect to the jet half-width $y_{\frac{1}{2}}$ are depicted in Fig. 4 and at streamwise station $\zeta =$ 22.032, the mean velocity profile differs insignificantly from

the Goertler solution, indicating that self-preservation in mean velocity profile is attained.

The variation of the non-dimensional centre-line velocity with streamwise distance is plotted in Fig. 5. The results agree well with the experimental data of Lai and Simmons (1979) and Zijnen (1958). The variation of the jet half-width with streamwise distance is compared with the result of Kotsovinos (1976) and Lai and Simmons (1979) in Fig. 6 and is found to give reasonably good agreement for streamwise distance less than about 40 nozzle widths. The discrepancy which exists at larger streamwise distance is not as significant as it appears because the results of Kotsovinos (1976) were obtained by fitting a third-order polynomial through the scattered data in the literature and most of the experimental data were not available for streamwise distance larger than 50 nozzle widths. Furthermore, uncertainity in the hot-wire data increases as the streamwise distance increases because of the decay of the velocity profile and possible three-dimensional and reversed flow effects. The rate of decay of the centre-line velocity $d\,\bar{U}_c^2/d(x/h)$ of 0.165 and the spreading rate $d(y_{\frac{1}{2}}/h)/d(x/h)$ of 0.106 obtained from Figs. 5 and 6 respectively fall within the range of values reported in the literature. As pointed out by Goldschmidt and Bradshaw (1980), exact self-preservation in mean quantities requires the kinematic virtual origin, obtained from the centre-line velocity decay curve, be equal to the geometric virtual origin obtained from the jet spreading curve. However, most

experimental data do not confirm this. In this study, the computed kinematic virtual origin agrees with the geometric virtual origin and is found to be 0.8h.

The development of the normalized shear stress profiles is shown in Fig. 7. The agreement between the computed shear stress profile at $\zeta = 95.032$ and the experimental results of Gutmark and Wygnanski (1976) is good considering that the shear stress term is the most difficult to be measured with sufficient accuracy and a scatter of at least 20% exists in the available experimental data in the literature. The maximum value of the computed non-dimensional shear stress term $\overline{u'v'}/U_c^2$ is 0.023 which agrees well with most measured values reported in Rodi (1975).

In jet computations, numerical errors introduced at the jet edge normally have little overall influence on the calculations as noted by McGuirk and Rodi (1979). The momentum integral in this study varies by less than 1% over a streamwise distance of 100 nozzle widths.

4.2 Unsteady Jet

The constant eddy-viscosity model which was found to give good agreement with experimental data was used in the computation of the preiodically pulsed jet. Results were obtained for three frequencies of pulsation, $\omega = 0.000871$, 0.00871 and 0.0871 which for a jet exit Reynolds number of 10^4 and a jet width h of 5mm correspond to 1, 10 and 100 Hz respectively. Two values of amplitude of pulsation were studied, namely $\varepsilon = 0.1$ and 0.15. The results were compared with the experimental data of Lai and Simmons (1979) and solutions for steady jets. The grid-sizes used in the streamwise (ζ) and transverse (η) directions are the same as those of the steady jet given in section 4.1.

4.2.1 Sensitivity of the Solution to the Convergence Criterion

Results show that solutions are very sensitive to the convergence criterion applied to points very near to the edge of the jet. Normally, the closer the point η_{con} is to the edge of the jet, the more difficult it will be for the solution to converge. It was found by Lai and Simmons (1979) that the convergence criterion can be applied to a point in the jet which varies with streamwise distance and is defined by

$$\eta_{con} = \begin{cases} (\eta_\infty)_i - 3 & \text{for } \zeta \leq 0.1 \\ 6(\eta_\infty)_{min}/10-1 & \text{for } \zeta \geq 0.1 \end{cases} \qquad (55)$$

This allows a comparatively sensitive measure of the convergence of the solution and yet maintains a reasonably

fast convergence without affecting the overall accuracy of the solution.

A fixed point arbitrarily defined by $\eta_{con} = \eta_{\infty} - 6$ may also be sufficient.

4.2.2 Validity of the Criterion for the Attainment of Steady State Solution

The criterion for the attainment of steady state solution described in section 3.6 is found to be adequate. The point η_s to which the steady state tests are applied has been chosen to coincide with η_{con} specified in equation (53).

4.2.3 Sensitivity of the Solution to Time Step

The sensitivity of the solution to time step has been tested for various frequencies. Calculations have been carried out with the period of pulsation divided into 12, and 49 intervals respectively. Results indicate that although division of a period into 12 time intervals seems to be too coarse, they differ insignificantly from each other. It is therefore sufficient to use 12 time intervals in the present study.

4.2.4 Relaxation of the Criterion for Jet Spreading

As pointed out in section 3.5, the jet spreads rapidly initially due to a steep velocity gradient. As the computations proceed downstream, the velocity profile becomes wider with a long tail at the jet edge. Because of the very low velocities at the jet edge, small numerical errors introduced there might cause instabilities and the criteria set out in equation (37) for the definition of the jet edge will not be satisfied resulting in continuous addition of unnecessary grid points. Futhermore, the number n_g of grid points which can

be added each time imposes a trade-off between the accuracy of the solution and the computing time required. This is because less computing time will be required if more points are added but since the quantities at the new points are obtained by extrapolation and the jet edge is particularly sensitive to small numerical errors, instability or inaccurate solution may result. The program provides choices of two different number of added grid points as the calculations are marched downstream and if the jet spreads beyond a certain η value which can be obtained from the Goertler solution, a new definition for the jet edge is applied to terminate the jet at the point where the velocity starts increasing or changes sign.

4.2.5 Results

Non-dimensional mean and instantaneous velocity profiles at various time delay intervals are plotted in Fig. 8 for the various frequencies and amplitudes of pulsation and $\zeta = 40.032$. The mean velocity profiles follow closely the steady jet curve. The instantaneous profiles also collapse into the steady jet curve except in Figs. 8(e) and (f) where a slight discrepancy occurs for $\omega = 0.00871$ and 0.0871 and $\varepsilon = 0.15$. This agrees with the trend observed in the experimental data of Lai and Simmons (1979), in which a slight departure of the non-dimensional instantaneous velocity profiles from the steady jet curve exists.

The mean centre-line velocity $\bar{U}_c$ obtained by time-averaging the steady state instantaneous centre-line velocity U_c over a period is shown in Fig. 9 to agree well with the centre-line velocity

decay curve for the steady jets for all the computed cases. Fig. 10 shows the variation of the jet half-width of the mean velocity profile with streamwise distance. For all cases, the mean jet half-width of the unsteady jet collapses on the steady jet spreading curve. A plot of Q/Q_E, a measure of mean entrainment, versus ζ for various frequencies and amplitudes of pulsation indicates that the mean entrainment rate does not differ from that of the steady jet, which is consistent with the trend associated with the mean centre-line velocity decay and jet spreading. In all the computed cases, the mean momentum flux M is conserved to within 1% over a streamwise distance of 100 nozzle widths, thus confirming the accuracy of the solutions.

With reference to Fig. 12, the non-dimensional mean shear stress profiles of the unsteady jet at $\zeta = 40.032$ agrees very well with that of the steady jet. The non-dinensional instantaneous shear stress profiles can also be collapsed into the steady jet profile except for $\omega = 0.0871$ in Figs. 12(c) and (f) where significant discrepancies are noted. This is consistent with the behavior of the instantaneous velocity profiles where a slight disagreement will be amplified in the velocity gradient term which is shown up in the shear stress profile.

Fig. 13 shows the variation of the steady-state instantaneous centre-line velocity with time for various frequencies and amplitudes of pulsation at three different streamwise distances. The reference signal is given by

$\bar{U}_c$ (1 + $\varepsilon \sin \omega t$). It can be noted that as the frequency and amplitude increases and for large streamwise distance, distortions in the waveform of the centre-line velocity exist, an indication of the importance of higher harmonics. Such a trend was also observed by Lai and Simmons (1979) in their experimentally pulsed jet. The variation of the phase angle ϕ between the steady-state fundamental component of the centre-line velocity at any streamwise station and that at the nozzle exit with ζ is depicted in Fig. 14. The phase angle is a lag which increases with both frequency and streamwise distance. However, the amplitude of pulsation does not have any significant effect on the phase lag variation. For a region sufficiently close to the nozzle, the flow varies with time in a quasi-steady manner. That is, the solution over the region of interest can be approximated by a sequence of steady jet solutions, each of which corresponds to the instantaneous conditions at the nozzle. The region over which quasi-steady approximations can be applied to an unsteady jet can readily be determined by using the phase plot in Fig. 14. It is taken arbitrarily here that a phase lag of 5 degrees defines the downstream limit to quasi-steady solutions. This definition enables the quasi-steady approximations to U_c to be accurate to within 1%. From Fig. 14 it is apparent that the region over which quasi-steady approximations can be used decreases with increasing frequency. The quasi-steady region for the computed frequencies and for ε = 0.1 or 0.15 is as follows -

$\omega = 0.000871 \qquad 0 \leq \zeta \leq 60$

$\omega = 0.00871 \qquad 0 \leq \zeta \leq 10$

$\omega = 0.0871 \qquad 0 \leq \zeta \leq 1$

The variation of the percentage peak-to-peak oscillation, F, of the steady-state instantaneous centre-line velocity expressed as a percentage of the mean value with streamwise distance is plotted in Fig. 15. Although for $\omega = 0.0871$, F differs quite significantly from its initial value of 20% or 30%, it varies very little for other tested frequencies and amplitudes of pulsation. In the experimental data of Lai and Simmons (1979), it was shown that for frequencies between 1 and 10 Hz, which correspond to $\omega = 0.000871$ and 0.00871 here, the general trend indicates that F increases to a maximum at 10 nozzle widths and drops off beyond that point and F increases by almost 40% over the initial value even for 1Hz case. The present computations using the constant eddy viscosity model fail to predict this trend.

5.0 Concluding Remarks

A method which employs a transformation to solve the thin shear layer equations for the steady and unsteady, two-dimensional turbulent free jet issuing into stationary air has been presented. A Prandtl type constant eddy viscosity formulation was used to model the Reynolds shear stress term. The transformation reduces the rate of spread of the jet and enables the use of variable grid sizes over predetermined regions leading to accurate predictions.

In the steady jet, the mean velocity profiles, the mean centre-line velocity decay rate, the spreading rate of the jet and the normalized shear stress profiles all show good agreement with experimental data.

In the unsteady jet, the mean flow characteristics, such as the mean velocity and shear stress profiles, the mean centre-line velocity decay, the mean jet spreading and entrainment all follow closely those of the steady jet for the range of tested frequencies and amplitudes of pulsation. This is in agreement with the only available experimental data of this type which are available in Lai & Simmons (1979). However, unsteady effects are apparent in the distortion of the waveform for the variation of the instantaneous centre-line velocity with time and in the increase in the phase lag with streamwise distance. For high frequencies and amplitude of pulsation or at large streamwise distance from the nozzle, instantaneous quantities depart from quasi-steady values. The flow region over which quasi-steady approximations are applicable has been established for various frequencies.

Although the constant-eddy viscosity model used gives good predictions for the steady jet and the mean quantities for the unsteady jet, it fails to predict accurately the instantaneous quantities especially the precentage peak to peak oscillation of the steady-state instantaneous centre-line velocity. This suggests that if only mean flow quantities are required and if the frequencies and amplitudes of pulsation are of the order of those in this study, the constant eddy-viscosity model is adequate. However, if instantaneous quantities are critical and if frequencies and amplitudes of pulsation are an order of magnitude higher than those in this study, a more refined turbulence model has to be sought and tested.

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Appendix A Structure and Listings of Computer Program

A.1 Sturcture of the Steady Jet Program

The structure of the input data is as follows:

CARD 1	NRG, (RG(I), I = 1, NRG), DEGR
FORMAT	I5, 6F10.6
CARD 2	PROD , CONST 1, CONST 2
FORMAT	3F10.6
CARD 3	CO, XO, AA, BB, NPG, INC, IFREQ, NXI
FORMAT	4F10.6, 4I4
CARD 4	(NXTT(I), I = 2, NXII)
FORMAT	10I4
CARD 5	(DELX(I), I=1,NXI), X(1)
FORMAT	8F10.6
CARD 6	(DETAC(I), I = 1,8)
FORMAT	8F10.6
CARD 7	(VC(I), I = 1,7), ETAE
FORMAT	8F10.6
CARD 8	E1, E2, E3
FORMAT	3F10.6
CARD 9	OUTDSK, LN
FORMAT	L1, I3
CARD 10	ETAG
FORMAT	F10.6

The symbols used have the following meaning:

NRG	No. of times grid points have to be rearranged
RG(I)	Upper η value above which grid points will not be dropped
DEGR	Lower η value below which grid points will not be dropped.
PROD	ν defined in section 2.1 (Jet Exit Reynolds Number)
CONST 1, CONST 2	c_1 and c_2 defined in Eq. (12)
CO	$\zeta_o^{\frac{1}{2}}/(ab)$
XO	ζ_o
AA	a defined in Eq. (15a)
BB	b defined in Eq. (15b)
NPG	n_ζ, No. of grid points to be added if the jet spreads
INC	interval, in terms of the number of transverse grid points, at which a value of the velocity profile is required
IFREQ	interval, in terms of the number of streamwise stations, at which velocity profile is required
NXI	No. of different grid sizes in streamwise direction
NXTT(I)	Streamwise station number at which a different grid size is used.
DELX (I)	Streamwise grid size $\Delta\zeta_i$
X(1)	Value of ζ at initial streamwise station (zero at nozzle exit)
DETAC(I)	Transverse grid size $\Delta\eta$
VC(I)	Values of η for each sub-region
ETAE	$(\eta_\infty)_i$
E1	Convergence limit defined in Eq. (36)
E2, E3	Jet edge definition defined in Eq. (37)

OUTDSK	Assumes logical value TRUE if solutions are written to disk
LN	Device number of writing to disk
ETAG	η_a defined in Eq. (28)

```
C
C**** MAIN PROGRAM
C**** THIS SOLVES THE THIN SHEAR LAYER(TSL) EQUATIONS IN NONDIMENSIONAL
C**** TRANSFORMED FORM FOR STEADY TWO DIMENSIONAL TURBULENT FREE JET BY
C**** KELLER BOX METHOD.
C
      COMMON/BL0/LC,LP
      COMMON/BL1/NP,NX,NXT,ITAU,VGP,DELTX,CNU,R,ETA(140),X(140),ETA(140
     1),IFLAG,INC
      COMMON/BL2/X(100)
      COMMON/BL3/DELG(140),F(140,2),G(140,2),Q(140,2),DELQ
      COMMON/BL5/E1,E2,E3
      COMMON/BL6/IT
      COMMON/BL13/NPPP,NP1
      COMMON/BL15/TN1(140,2)
      COMMON/BL21/CLIPSK,LP
      COMMON/BL40/CC(5),IC,NPG,PICO
      DIMENSION DELG(140)
      LOGICAL CLIPSK
C     ---------------------------------
      LC=5
      LP=6
      ITMAX=50
      IC=1
      NX=1
      WRITE(LP,900)
      CALL INPUT
      CALL GRID
      CALL IVPL
      NPPP=NP-1
      NP1=NP-1
      NPM=NP1
      G(1,1)=G(1,2)
   50 CONTINUE
      IF(NX.LT.NXT)GO TO 60
      WRITE(LP,670)
      STOP 10
   60 CONTINUE
      CALL TCPY
      WRITE(LP,900)NX,X(NX)
      IT=0
      IF(NX.EQ.1)GO TO 5000
  100 IT=IT+1
      NP1=NP-1
      IF(IT.LT.ITMAX)GO TO 120
      DO 615 J=1,NP,INC
  615 WRITE (LP,660) J,ETA(J),F(J,2),G(J,2),Q(J,2),DELG(J)
      WRITE (LP,910)
      STOP 20
  120 CONTINUE
C**** SOLVING THE LINEARISED MOMENTUM EQUATION
      CALL COEFF
      CALL SOLV
C**** CHECK FOR CONVERGENCE
      TEST=DELG(NP1)/G(NP1,2)
      TESTM=DELG(NPM)/G(NPM,2)
      TESTQ=DELG(NP1)/G(NP1,2)
      IF(ABS(TEST).GT.E1.OR.ABS(TESTM).GT.E1.OR.ABS(TESTQ).GT.E1)
     1GO TO 100
C**** CHECK FOR GROWTH
      IF(NP.GT.140)GO TO 320
      NPM=NP1
      TEST1=G(NP,2)
      TEST2=G(NP-1,2)/G(1,2)
      IF(ABS(TEST1).LT.E2.AND.ABS(TEST2).LT.E3)GO TO 320
      CALL GROWTH
      CALL TCPY
      GO TO 100
  320 CONTINUE
      COMP=TNL(NP1,2)
      CALL TCPY
      CONVT=(TNL(NP1,2)-COMP)/TNL(NP1,2)
      IF(ABS(CONVT).LE.0.01)GO TO 5000
      GO TO 100
```

```
 5100 CONTINUE
      IF(IO.GT.NSO)GO TO 5010
      IF(XT(NS).LT.SO(IO))GO TO 5010
      CALL [illegible]
      IO=IO+1
 5010 CONTINUE
      WRITE (LO,3300)IT,C(1,2),D LS(1)
      CALL [illegible]
      GO TO [illegible]
C     ------------------------------------
    4 FORMAT(1H1////1X,75H[illegible] COMPUTATION OF STEADY TURBULENT 2-D
     1[illegible] JET BY [illegible]///)
  900 FORMAT(///5X,6H[illegible] = ,I4,[illegible]X,4H[illegible] = ,E12.5)
  [illegible] FORMAT(///1X,[illegible] I[illegible]X)
 3300 FORMAT(5X,[illegible] = ,I[illegible],[illegible]X,[illegible] = ,E14.[illegible],5X,8H[illegible]LOS = ,E14.6)
 [illegible] FORMAT([illegible],[illegible]),6,4X,[illegible](6X,1[illegible].7))
 [illegible] FORMAT(///10X,27H[illegible] OF CALCULATIONS ***)
      END

      SUBROUTINE INPUT
      COMMON/BLC/LC,LO
      COMMON/B[illegible]/[illegible],[illegible]XT,[illegible],VCP,[illegible]X,[illegible],D,DETA(140),[illegible](140),ETA(140
     1),[illegible],INC
      COMMON/BL3/X(100)
      COMMON/BL5/[illegible]
      COMMON/BL7/NPC([illegible]),V([illegible]),[illegible]TAC(8)
      COMMON/BL9/[illegible]
      COMMON/BL1[illegible]/[illegible]
      COMMON/BL2[illegible]/[illegible]
      COMMON/BL2[illegible]/NPC
      COMMON/BL31/[illegible]K,[illegible]
      COMMON/BL[illegible]/[illegible](5),[illegible]
      COMMON/BL[illegible]/[illegible]
      DIMENSION NXTI([illegible]),[illegible]Y(7)
      LOGICAL [illegible]
C     ------------------------------------
C**** THIS READS IN THE INPUT DATA.
C
      READ(LC,1400)NPC,(-[illegible]),I=1,NPC),D=GP
      READ(LC,1300)[illegible]
      READ(LC,1[illegible])[illegible],INC,IPRNO,NXT
      NXTI=NXT+1
      READ(LC,1000)(NXTT(I),I=1,NXTI)
      NXT=[illegible](NXTI)
      READ(LC,1100)([illegible](I),I=1,NXT),X(1)
      READ(LC,1100)([illegible](I),I=1,[illegible])
      READ(LC,1[illegible])(V(I),I=1,7),[illegible]
      READ(LC,1[illegible])[illegible]
      READ(LC,1[illegible])[illegible]
      READ(LC,1[illegible])[illegible]
      NXTI(1)=1
      DO 900 I=1,NXT
  900 CALL [illegible](NXTT(I),NXTT(I+1))
      WRITE(LO,1300)[illegible],[illegible],DPGR
      WRITE (LO,17[illegible])[illegible]
      WRITE (LO,1[illegible])[illegible],INC,IPRIC,NXT
      WRITE (LO,1[illegible])[illegible]
      WRITE (LO,1[illegible])[illegible]
      WRITE (LO,1[illegible])[illegible]
      WRITE (LO,1[illegible])(V(I),I=1,7),[illegible]
      WRITE (LO,1[illegible])[illegible]
      WRITE (LO,1[illegible])[illegible]
      RETURN
C     ------------------------------------
    1 FORMAT(F1[illegible].6)
 1000 FORMAT([illegible]4)
 1100 FORMAT([illegible]10.4)
 1[illegible] FORMAT([illegible]1[illegible],4I[illegible])
 1[illegible] FORMAT([illegible]10.4)
 1[illegible] FORMAT([illegible])
 1700 FORMAT([illegible])
 1[illegible] FORMAT(1[illegible],[illegible]1[illegible].6)
 17[illegible] FORMAT([illegible])
 1[illegible] FORMAT(5X,[illegible])
```

```
1650 FORMAT(5X,ET4)
1520 FORMAT(5X,5(F10.4,3X))
1950 FORMAT(5X,3(F10.0,2X))
     END

     SUBROUTINE CALX(DELX,NXTI,NXTF)
     COMMON/BL1/X(100)
     DO 900 I=NXTI,NXTF
 900 X(I+1)=X(I)+DELX
     RETURN
     END

     SUBROUTINE OUTPUT
     COMMON/BL0/LC,LP
     COMMON/BL1/NP,NX,NXT,ETAL,VGP,DELTX,CNU,D,DETA(140),A(140),ETA(140
    1),IFREQ,INC
     COMMON/BL2/X(100)
     COMMON/BL3/DELC(140),F(140,2),G(140,2),C(140,2)
     COMMON/BL6/E1,E2,E3
     COMMON/BL9/XC,C1,C4,C5
     COMMON/BL12/NPHH,NP1
     COMMON/BL13/TNU(140,2)
     COMMON/BL14/GNOR(140),GHALF
     COMMON/BL15/NPC
     COMMON/BL51/OUTDSK,LN
     COMMON/BL103/UC
     LOGICAL OUTDSK
     DATA IT/0/
C    ------------------------------------
C*** THIS OUTPUTS ALL THE REQUIRED QUANTITIES
C
     GMAX=G(1,2)
     DO 6100 J=1,NP
6100 GNOR(J)=G(J,2)/GMAX
     DO 6111 I=1,NP
     IF(GNOR(I).LT.0.5)GO TO 6112
6111 CONTINUE
6112 GHALF=ETA(I-1)+DETA(I-1)*(GNOR(I-1)-0.5)/(GNOR(I-1)-GNOR(I))
     UC=SQRT(XC)*G(1,2)/SQRT(X(NX)+XC)
     WRITE(LP,6450)UC
     CALL FILL
     DO 6250 J=NPHH,NP1
     IF(ABS(G(J+1,2)).LT.E2.AND.ABS(G(J,2)/G(1,2)).LT.E3)GO TO 6260
6250 CONTINUE
6260 NP=J+1
     WRITE(LP,6800)NP
     WRITE(LP,6900)GHALF
C*** SHIFT PROFILES
     DO 6300 J=1,NP
     TNU(J,1)=TNU(J,2)
     F(J,1)=F(J,2)
     G(J,1)=G(J,2)
6300 C(J,1)=C(J,2)
     IF(NX.EQ.1)GO TO 6120
     IT=IT+1
     IF(IT.NE.IFREQ)GO TO 6190
     IT=0
6120 WRITE(LP,6500)
     DO 6150 J=1,NP,INC
6150 WRITE(LP,6600)J,ETA(J),F(J,2),G(J,2),C(J,2),GNOR(J),DELC(J)
6190 CONTINUE
     IF(OUTDSK)WRITE(LN)NX,NP,X(NX),(F(J,2),J=1,NP),(G(J,2),J=1,NP),
    1                       (C(J,2),J=1,NP),(TNU(J,2),J=1,NP),
    2                       (ETA(J),J=1,NP)
     NX=NX+1
     RETURN
C    ------------------------------------
6450 FORMAT(/10X,'CENTERLINE VEL. UC =',F7.4)
6500 FORMAT(////4X,1HJ,6X,3HETA,13X,1HF,13X,1HG,23X,1HC,23X,4HGNOR///)
6600 FORMAT(I5,F11.5,4X,E16.7,4(6X,E16.7))
6700 FORMAT(///10X,27H*** END OF CALCULATIONS ***)
6800 FORMAT(10X,33HNO. OF TRANSVERSE GRID POINTS, NP = ,I4)
6900 FORMAT(10X,17HGHALF HALF WIDTH = ,F7.4)
     END
```

```
      SUBROUTINE [illegible]
      COMMON/BL1/NP,NX,NXT,[illegible],VGP,DELTX,CNU,C,DETA(140),A(140),ETA(140
     1),[illegible]
      COMMON/BL7/X(100)
      COMMON/BL3/[illegible]LC(140),F(140,2),G(140,2),C(140,2)
      COMMON/BL5/YC,[illegible]
      COMMON/BL1[illegible]/CONST1,CONST2
      COMMON/BL1[illegible]/FFFF,[illegible]
      COMMON/BL13/TNL(140,2)
      COMMON/BL2[illegible]/FREQ
      COMMON/BL1[illegible]/LC
C     ---------------------------------
      DO [illegible] I=1,NP1
      COMP1=G(I,2)/G(1,2)
      COMP2=G(I+1,2)/G(1,2)
      IF(ABS(COMP1).GT.0.50.AND.ABS(COMP2).LE.0.50)GO TO 2010
 2000 CONTINUE
 2010 WIDTH1=ETA(I)+(ABS(COMP1)-0.5)*DETA(I)/(ABS(COMP1)-ABS(COMP2))
      [illegible]=X([illegible])+[illegible]
      CONST=CONST1
      IF(LC.LT.[illegible])CONST=CONST2
      TCONST=CONST*[illegible](1,2)*SQRT([illegible])*WIDTH1+FREQ
      DO [illegible] I=1,NP
 2050 TNL(I,2)=TCONST
      RETURN
C     ---------------------------------
      END

      SUBROUTINE IVEL
      COMMON/BL[illegible]/LC,LP
      COMMON/BL1/NP,NX,NXT,[illegible],VGP,DELTX,CNU,C,DETA(140),A(140),ETA(140
     1),[illegible]
      COMMON/BL3/[illegible]LC(140),F(140,2),C(140,2),G(140,2)
      COMMON/BL7/NPC(8),VC(8),[illegible]TAC(8)
      COMMON/BL5/YC,CC
      COMMON/BL13/TNL(140,2)
      COMMON/BL2[illegible]/NFC
      COMMON/BL1[illegible]/[illegible]TAG
C     ---------------------------------
C**** THIS SETS UP THE INITIAL VELOCITY PROFILE
C
      DO 1[illegible] J=1,NP
      IF([illegible](J).GT.[illegible])GO TO 190
 180  CONTINUE
 190  JJ=J-1
      F(1,2)=0.
      C(1,2)=CC
      G(1,2)=0.
      DO 200 J=2,JJ
      C(J,2)=CC
      F(J,2)=[illegible](J)*C(J,2)
 200  G(J,2)=0.
      [illegible]=JJ+1
      CC1=[illegible]-ETA(JJ)
      DO 300 J=[illegible],NP
      CC=ETA(J)-ETA(JJ)
      ETAD=CC/CC1
      F(J,2)=F(JJ,2)+CC*CC1*ETAD*(1.-ETAD*ETAD+0.5*ETAD*ETAD*ETAD)
      C(J,2)=CC*(1.-ETAD)*(1.-ETAD)*(1.+2.*ETAD)
 300  G(J,2)=-6.*CC*ETAD*(1.-ETAD)/CC1
      RETURN
C     ---------------------------------
      END

      SUBROUTINE GRID
      COMMON/BL[illegible]/LC,LP
      COMMON/BL1/NP,NX,NXT,[illegible],VGP,DELTX,CNU,C,DETA(140),A(140),ETA(140
     1),[illegible]
      COMMON/BL7/NPC(8),VC(8),[illegible]TAC(8)
C     ---------------------------------
C**** WITH CORRECTING IVAL, THIS SETS UP A NETWORK OF VARIABLE GRID SIZE
C
      ETA(1)=0.
```

```
      NPP=2
      NPC(1)=VC(1)/DETAC(1)+1.
      DO100 I=1,7
      IF(I.EQ.1)GO TO 50
      NPC(I)=NPC(I-1)+IFIX((VC(I)-VC(I-1))/DETAC(I)+0.5)
   50 IF(VC(I).EQ. 0.)L=I
      IF(NPC(I).GT.140)NPC(I)=140
      CALL EVAL(NPP,NPC(I),VC(I),DETAC(I))
      NPP=NPC(I)+1
      IF(NPP.GT.140)GO TO 1700
  100 CONTINUE
      VC(8)=VC(7)+DETAC(8)*FLOAT(140-NPC(7))
      CALL EVAL(NPP,140,VC(8),DETAC(8))
 1700 CONTINUE
      NP=NPC(L)
      IF(NP.GT.140)WRITE(LP,1800)ETA(140)
      RETURN
C     ------------------------------------
 1800 FORMAT(10X,23H*** NP EXCEEDED 140 ***,5X,11HETA(140) = ,F10.6)
      END

      SUBROUTINE REGRID
      COMMON/BL0/LC,LP
      COMMON/BL1/NP,NX,NXT,ETAI,VGP,DELTX,CHU,P,DETA(140),A(140),ETA(140
     1),IF,DO,INC
      COMMON/BL2/P LC(140),F(140,2),C(140,2),G(140,2),DELG
      COMMON/BL7/NPC(8),VC(8),DETAC(8)
      COMMON/BL13/INL(140,2)
      COMMON/BL47/RC(5),IC,NRG,DREG
C     ------------------------------------
C**** THIS RE-SETS THE GRID SYSTEM BY DROPPING SPECIFIED POINTS
C
      WRITE(LP,110)
      DO50 I=1,NP
      IF(ETA(I).NE.DREG)GO TO 50
      INC=I
      GO TO 60
   50 CONTINUE
   60 CONTINUE
      INI=INC+1
      DO30 I=1,8
      IF(VC(I).LT.RG(IC))GO TO 30
      IVC=I-1
      IF(I.EQ.NPC)IVC=I
      GO TO 40
   30 CONTINUE
   40 CONTINUE
      DECR=VC(IVC)
      DO70 I=INI,NP
      IC=INC+2*(I-INC)
      IF(ETA(IC).LT.VC(IVC))GO TO 65
      IF(ETA(IC-2).NE.VC(IVC))GO TO 68
      INJ=I-1
      ICC=IC-2
      GO TO 80
   68 INJ=I
      ICC=IC-1
      GO TO 80
   65 CONTINUE
      F(I,2)=F(IC,2)
      C(I,2)=C(IC,2)
      G(I,2)=G(IC,2)
      INL(I,2)=INL(IC,2)
      ETA(I)=ETA(IC)
      DETA(I-1)=ETA(I)-ETA(I-1)
      A(I)=0.5*DETA(I-1)
   70 CONTINUE
   80 IF(ETA(NP).GT.VC(IVC))NPN=INJ+NP-ICC
      IF(ETA(NP).LT.VC(IVC))NPN=NP-(NP-INC+1)/2
      IF(ETA(NP).EQ.VC(IVC))NPN=INJ
      NP=NPN
      IF(NP.LT.INJ)GO TO 1000
      DO90 I=INJ,NP
      IC=ICC+I-INJ
```

```
      F(I,2)=F(IC,2)
      G(I,1)=G(IC,1)
      G(I,2)=G(IC,2)
      TAL(I,2)=TAL(IC,2)
      ETA(I)=ETA(IC)
      DETA(I-1)=ETA(I)-ETA(I-1)
      A(I)=0.5*DETA(I-1)
   95 CONTINUE
  100 CONTINUE
      NPC(IVC)=INJ
      NPP=IN J+1
      IVC1=IVC+1
      IF(IVC1.GT.7)GO TO 105
      DO 100 I=IVC1,7
      NPC(I)=NPC(I-1)+[illegible](I)- TA(NPP-1))/DETAC(I)+0.5)
      IF(NPC(I).GT.140)NPC(I)=140
      CALL [illegible]VAL(NPP,NPC(I),A(I),DETAC(I))
      NPP=NPC(I)+1
      IF(NPP.GT.140)[illegible]
  100 CONTINUE
  105 CONTINUE
      VC(8)=VC(7)+DETAC(8)*[illegible](140-NPC(7))
      CALL [illegible]VAL(NPP,1[illegible],VC( ), [illegible](8))
      DO 120 I=1,140
  120 WRITE(LP,130)I,ETA(I),[illegible](I),A(I)
      RETURN
C     ---------------------------------------
  110 FORMAT(1X,[illegible] [illegible] RESTRUCTURED *****)
  130 FORMAT(I5,3(2X,[illegible].F))
      END

      SUBROUTINE [illegible]VAL([illegible])
      COMMON /[illegible]/[illegible],[illegible],VC[illegible],DELTX,CNL,D,DETA(140),A(140),ETA(140
     1),[illegible]
C     ---------------------------------------
      DO 175 J=NPP,[illegible]
      DETA(J-1)=[illegible]
      A(J)=0.5*DETA(J-1)
      ETA(J)=ETA(J-1)+DETA(J-1)
 1750 CONTINUE
      IF(NPP.EQ.140)[illegible]
      [illegible]=(ETA(NPP)-[illegible])/[illegible]
      IF([illegible].GT.[illegible])GO TO 1760
      ETA(NPP)=E
      RETURN
 1760 NPP=NPP+1
      ETA(NPP)=E
      DETA(NPP-1)=ETA(NPP)-ETA(NPP-1)
      A(NPP)=0.5*DETA(NPP-1)
      RETURN
C     ---------------------------------------
      END

      SUBROUTINE [illegible]
      COMMON /[illegible]/[illegible],XT,[illegible],DELTX,CNL,D,DETA(140),A(140),ETA(140
     1),[illegible]
      COMMON /[illegible]/X([illegible])
      COMMON /[illegible]/[illegible],G(140,2),G(140,2)
      COMMON /[illegible]/[illegible],S4(140),S5(140),S6(140),
     1[illegible]
      COMMON /[illegible]/[illegible]
      COMMON /[illegible]/[illegible]
C     ---------------------------------------
C**** THESE [illegible] THE [illegible] OF THE LINEARIZED SYSTEM RESULTING
C**** [illegible]
C
      [illegible]=(X(NX)+X(NX-1)+[illegible])/(X(NX)-X(NX-1))
      [illegible]
      [illegible]
      [illegible]
      [illegible]=X(NX-1)+[illegible]
      [illegible]
      [illegible]
      DO 250 J=2,[illegible]
```

```
C**** PRESENT STATION
      CFB=0.5*(F(J,2)*G(J,2)+F(J-1,2)*G(J-1,2))
      FGB=0.5*(F(J,2)*G(J,2)+F(J-1,2)*G(J-1,2))
      FB=0.5*(F(J,2)+F(J-1,2))
      GB=0.5*(G(J,2)+G(J-1,2))
      GGB=0.5*(G(J,2)*G(J-1,2))
C**** PREVIOUS STATION
 2100 FB=0.5*(F(J,1)+F(J-1,1))
      GB=0.5*(G(J,1)+G(J-1,1))
      CFC=0.5*(F(J,1)*G(J,1)+F(J-1,1)*G(J-1,1))
      GGS=0.5*(G(J,1)*G(J,1)+G(J-1,1)*G(J-1,1))
C**** COEFFICIENTS OF THE DIFFERENCED MOMENTUM EQUATION
      DLTST=1./DLT2(J-1)
      S1(J)=TAU(J,2)*DLTST+0.5*ALP1*F(J,2)-0.5*ALP*CFB
      S2(J)=-TAU(J-1,2)*DLTST+0.5*ALP1*F(J-1,2)-0.5*ALP*CFB
      S3(J)=0.5*ALP1*G(J,2)+0.5*ALP*CGB
      S4(J)=0.5*ALP1*G(J-1,2)+0.5*ALP*CGB
      S5(J)=ALP1*G(J,2)
      S6(J)=ALP1*G(J-1,2)
C**** DEFINITIONS OF R J
      CLB=DLTST*(TAU(J,1)*G(J,1)-TAU(J-1,1)*G(J-1,1))+CGSB+CFCB
      CRB=-CLB+ALP*(CFB-CGGB)
      R1(J)=F(J-1,2)-F(J,2)+DLT(J-1)*GB
      R2(J)=CRB-DLTST*(TAU(J,2)*G(J,2)-TAU(J-1,2)*G(J-1,2))-ALP1*FGB
     1      -ALP*CFB-ALP*(CGGB-CFB*GB)
      R3(J-1)=G(J-1,2)-G(J,2)+DLTA(J-1)*GB
 2500 CONTINUE
      R1(1)=0.
      R2(1)=0.
      R3(NP)=0.
      RETURN
C     ----------------------------------
      END

      SUBROUTINE SOLV
      DIMENSION W1(140),W2(140),W3(140),A11(140),A12(140),A13(140),
     1          A21(140),A22(140),A23(140),G11(140),G12(140),G13(140),
     2          G21(140),G22(140),G23(140),DTLG(140),DELF(140)
      COMMON/BLC/LC,LP
      COMMON/BL1/NP,NX,NXT,TAT,VCP,DELTA,CNU,D,DETA(140),A(140),ETA(140
     1),IFRSC,TAC
      COMMON/BL3/DTLG(140),F(140,2),G(140,2),C(140,2),CELC
      COMMON/BL4/S1(140),S2(140),S3(140),S4(140),S5(140),S6(140),
     1R1(140),R2(140),R3(140)
      COMMON/BL6/IT
C     ----------------------------------
C**** THIS SOLVES THE LINEARIZED SYSTEM BY THE BLOCK ELIMINATION METHOD.
C
C**** W ELEMENTS FOR J=1
      W1(1)=R1(1)
      W2(1)=R2(1)
      W3(1)=R3(1)
C**** ALPHA ELEMENTS FOR J=1
      A11(1)=1.
      A12(1)=0.
      A13(1)=0.
      A21(1)=0.
      A22(1)=0.
      A23(1)=1.
C**** GAMMA ELEMENTS FOR J=2
      G11(2)=-1.
      G12(2)=A(2)*A(2)
      G13(2)=A(2)
      G21(2)=S4(2)
      G22(2)=S2(2)-A(2)*S6(2)
      G23(2)=-S6(2)
C**** FORWARD SWEEP
      DO 3000 J=2,NP
      IF(J.EQ.2)GO TO 3000
      DEN=A13(J-1)*A21(J-1)-A23(J-1)*A11(J-1)-A(J)*(A12(J-1)*A21(J-1)-
     1A22(J-1)*A11(J-1))
      DEN1=A22(J-1)*A(J)-A23(J-1)
      G11(J)=(A23(J-1)+A(J)*(A(J)*A21(J-1)-A22(J-1)))/DEN
      G12(J)=-(A(J)*A(J)+G11(J)*(A12(J-1)*A(J)-A13(J-1)))/DEN1
```

```
      SUF(NX)=AVF(NX)+SQRT(X(NX)+X2)
      IF(NX.[illegible].1)GO TO 5950
      [illegible]
      [illegible]
 5950 [illegible]
      [illegible]
      WRITE (LP,5100)[illegible],AMF(NX),ACD
      [illegible](NX)=AMT/1[illegible]
      [illegible](NX)=ACD/1CC.
      RETURN
C     --------------------------------------
 5100 FORMAT(/10X,13HAV[illegible] = ,E15.7,7X,7HV/V[illegible] = ,F6.2,
     1       [illegible] C[illegible]/1 X,15HAV[illegible] MASS [illegible] = ,E15.7,5X,7HC/C[illegible] =
     2       ,F6.2,[illegible])
      END

      SUBROUTINE GROWTH
      COMMON/BLC/LC,LP
      COMMON/BL1/[illegible],NX,[illegible],VCP,DELTX,[illegible],DITA(140),A(140),[illegible](140
     1),[illegible],I[illegible]
      COMMON/[illegible]/[illegible](140),F(140,2),G(140,2),C(140,2)
      COMMON/BL[illegible]/TNL(140,2)
      COMMON/BL[illegible]/NPC
C     --------------------------------------
C**** THIS CHECKS FOR THE SPREADING OF THE JET.
C
      [illegible]
      NP1=NP+1
      NP=NP+[illegible]
C**** DEFINITION OF PROFILES FOR NEW NP
      [illegible]=[illegible](NP)-[illegible](NP1)
      DO 4050 J=NP1,NP
      TNL(J,1)=[illegible].
      TNL(J,2)=TNL(NP1,2)
      [illegible]=[illegible](J)-[illegible](NPC)
      DO 4050 I=1,2
      F(J,I)=[illegible](NPC,I)+F(NP1,I)
      C(J,I)=C(NPC,I)
      G(J,I)=[illegible].
 4050 CONTINUE
      WRITE (LP,41 )NPC
      RETURN
C     --------------------------------------
   41 FORMAT(//5X,12H[illegible] GROWTH-,I3,13H-POINTS ADDED)
      END
```

A.3 Structure of the Unsteady Jet Program

The structure of the input data is as follows:

CARD 1	NRG, (RG(I), I = 1, NRG), DEGR
FORMAT	I5, 6F10.6
CARD 2	(NXG(I), I - 1, NRG)
FORMAT	5I5
CARD 3	NPG, NPG2, START, LN, NEW, NTRANS, NPRINT
FORMAT	7I5
CARD 4	CONST 1, CONST 2
FORMAT	2F10.4
CARD 5	CO, XO, EPS, OMG, NT, INC
FORMAT	4F10.4, 2I5
CARD 6	IFREQ, IFR, IFA, NXI
FORMAT	4I5
CARD 7	(NXTT(I), I=2, NXII)
FORMAT	10I5
CARD 8	(DELX(I), I = 1, NXI), X(1)
FORMAT	8F10.4
CARD 9	(DETAC(I), I = 1,8)
FORMAT	8F10.4
CARD 10	(VC(I), I = 1.7), GTAG
FORMAT	8F10.4
CARD 11	E1, E2, E3, E4, E5, AA, BB, PROD
FORMAT	8F10.4
CARD 12	ETAG
FORMAT	F10.4

Symbols which appear also in the steady jet program have the same meanings here. All other symbols are defined as follows: -

NXG(I)	Streamwise station at which grid has to be rearranged.
NPG 2	Second choice of number of transverse grid points to be added if jet spreads as discussed in section 4.2.4
START	Streamwise station number at which computations start
NEW	See below
NTRANS	Streamwise station number at and beyond which constant c_2 is used.
NPRINT	Streamwise station number at and beyond which velocity profiles are required
EPS	ε, amplitude of pulsation
OMG	ω, angular frequency of pulsation
NT	Number of time intervals in a period + 1
IFR	interval, in terms of the number of streamwise stations, at which instantaneous velocity profiles are required.
IFA	interval, in terms of the number of streamwise stations, at which mean quantities are required
E4	ε_4, ε_5 and ε_7 defined in Eq. (40) and (41)
E5	ε_6 defined in Eq. (40)

Because of the initially steep velocity gradient, the velocity gradient curve behaves erratically at the region around the initial jet edge which constitutes about 3% of the total jet-width. To eliminate such numerical erratic behavior, a first-order smoothing function can be applied and is supplied through the Subroutine SMOOTH. The use of the

smoothing function does not affect the overall results and is optional. The parameter NEW specifies the streamwise station number where smoothing function starts to be used. If smoothing function is not required, NEW can assume the value of the final streamwise station number.

A.4 Listing of Unsteady Jet Program


```
C
C**** MAIN PROGRAM
C**** THIS SOLVES THE THIN SHEAR LAYER (TSL) EQUATIONS IN NONDIMENSIONAL
C**** TRANSFORMED FORM FOR UNSTEADY TWO-DIMENSIONAL TURBULENT FREE JET
C**** BY THE KELLER BOX METHOD.
      COMMON /BL0/LC,LP
      COMMON /BL1/[illegible],[illegible](51),DX,[illegible]XT,NT,DETA(1[illegible]),A(1[illegible]),ETA(1[illegible]),[illegible],
     1            [illegible]
      COMMON /BL2/X(100)
      COMMON /BL3/[illegible](1[illegible]),F(51,1[illegible],2),U(51,1[illegible],2),V(51,100,2)
      COMMON /BL4/INT,INT1,INT2,[illegible]MAX,NMAIN
      COMMON /BL5/ 1, 2, 3, 4,[illegible]
      COMMON /BL[illegible]/NCH
      COMMON /BL1[illegible]/[illegible](51)
      COMMON /BL1[illegible]/NT1
      COMMON /BL[illegible]/IN,[illegible]
      COMMON /BL[illegible]/T[illegible](51,100,2),[illegible]
      COMMON /BL[illegible]/[illegible]
      COMMON /BL[illegible]/[illegible](100)
      COMMON /BL[illegible]/[illegible]
      COMMON /BL[illegible]/NX
      COMMON /BL[illegible]/PG(5),IC,[illegible],[illegible],NXG(5)
      INTEGER START
      LOGICAL SS1,CHECK,LOGIC
C     ------------------------------------
      LC=5
      LP=6
      IC=1
      ITMAX=1[illegible]
      NX=1
      WRITE(LP,[illegible])
      CALL [illegible]
      CALL GRID
      NAPB=NP-3
      NPH=NAPB
      NPMAX=NP
      NPMIN=NP
      [illegible]=1.
C**** COMPUTE INITIAL CONDITIONS IN (ETA,T) PLANE
      CALL IVPL
      NTT=NT+1
      DO 10 INT=1,NT
   10 CALL COPY
      [illegible]
   20 IW=1
      NXX=NX
      IF(X(NX).GT.[illegible])NAPB=6*NPMIN/10-1
      IPT=0
      CHECK=.FALSE.
      IL=[illegible]
      IF(NX.EQ.1)GO TO 280
      INT=INT+1
      IF(INT.GT.NT1)INT=1
C**** COMPUTE INITIAL CONDITIONS IN (X,ETA) PLANE
      CALL INITCN
      WRITE(LP,[illegible])IW
      JJJ=JJJ+1
C**** MARCHING ALONG T-AXIS
      ICOUNT=1
      [illegible]=.FALSE.
   4[illegible] II=[illegible]
      INT=INT+1
      I[illegible]T=INT-1
      IF(INT.GT.NT1)INT=1
      IF(ICOUNT.LT.NT1)GO TO 55
      ICOUNT=[illegible]
      IW=IW+1
      IF(IW.GT.24)CALL EXIT
      [illegible]=.TRUE.
      WRITE(LP,[illegible])IW
   55 CONTINUE
C**** TO STORE PROFILES FOR CHECKING THE ATTAINMENT OF STEADY
```

```
C**** STATIC SOLUTION
      IF(.NOT.SS)GO TO 53
      INDEX=INT
      ICC=NT
      INK=INT
      NPA(NT)=NPA(INDEX)
      NPAS=NPA(INDEX)
      DO45 J=1,NPAS
      F(NT,J,2)=F(INDEX,J,2)
      G(NT,J,2)=G(INDEX,J,2)
   45 C(NT,J,2)=C(INDEX,J,2)
   53 CONTINUE
      ICOUNT=ICOUNT+1
      NF=NPAC(INT)
      IIN=INT
      IN=1
      IF(NPA(INTT).GE.NP)GO TO 190
      NF=NPA(INTT)
      IIN=INTT
      IN=2
  190 NPA(INT)=NPAC(INT)
      DO250 J=1,NF
      F(INT,J,2)=F(IIN,J,IN)
      G(INT,J,2)=G(IIN,J,IN)
      C(INT,J,2)=C(IIN,J,IN)
  250 TNU(INT,J,2)=TNU(IIN,J,IN)
      IF(NPAC(INTT).GE.NF)GO TO 100
      NPTD=NPAC(INTT)
      NPT=NPTD+1
      DO199 J=NPT,NP
      F(INTT,J,1)=F(INTT,NPTD,1)
      G(INTT,J,1)=G(INTT,NPTD,1)
      C(INTT,J,1)=0.
  199 TNU(INTT,J,1)=TNU(INTT,NPTD,1)
C**** SOLVING THE LINEARIZED MOMENTUM EQUATION
  100 IT=IT+1
      IF(IT.GT.ITMAX)GO TO 400
      CALL COEF
      CALL SOLV
C**** CHECK FOR CONVERGENCE
      TEST=DELG(NAPB)/G(INT,NAPB,2)
      TESTM=DELG(NP-6)/G(INT,NP-6,2)
      TESTO=DELG(NP-6)/G(INT,NP-6,2)
      IF(ABS(TEST).GT.E1.OR.ABS(TESTM).GT.E1.OR.ABS(TESTO).GT.E4)
     1 GO TO 100
C**** CHECK FOR GROWTH
      J1=NPAC(INT)-5
      DO196 J=J1,NF
      TESTG1=C(INT,J,2)
      TESTG2=C(INT,J,2)/G(INT,1,2)
      IF(ABS(TESTG1).GT.E2.OR.ABS(TESTG2).GT.E3)GO TO 196
      NPA(INT)=J
      GO TO 230
  196 CONTINUE
C**** RELAXING GROWTH CRITERION
      IF(ETA(NF).LT.0.25)GO TO 230
      NFF=NP
      DO225 J=41,NFF
      TESTG1=C(INT,J,2)
      TESTG2=C(INT,J,2)/G(INT,1,2)
      IF(ABS(TESTG1).GT.E2.OR.ABS(TESTG2).GT.E3)GO TO 225
      NPA(INT)=J
      GO TO 200
  225 CONTINUE
      DO227 J=41,NFF
      IF(ABS(G(INT,J,2)).GT.ABS(G(INT,J+1,2)).AND.
     1(G(INT,J,2)*G(INT,J+1,2)).GT.0.)GO TO 227
      NPA(INT)=J
      GO TO 200
  227 CONTINUE
  230 CONTINUE
      CALL GROWTH
      CALL PROY
      GO TO 100
```

```
  200 CONTINUE
      NF=NF1(INT)
      COMP=TAU(INT,NAPE,2)
      CALL CORY
      COMPR=(TAU(INT,NAPE,2)-COMPR)/TAU(INT,NAPE,2)
      IF(ABS(COMPR).GT.1.E-2)GO TO 100
C**** CHECK FOR STEADY STATE SOLUTION
      IF(JJJ.NE.IFREQ)GO TO 261
      
      CHECK=.TRUE.
  261 IF(CHECK)WRITE(LP,950)NX,X(NX),INT,IT,C(INT,1,2),DELC(1)
      IF(NX.GT.IT)WRITE (LP,950)NX,X(NX),INT,IT,C(INT,1,2),DELC(1)
      IF(.NOT.SST)GO TO 4
      TEST1=(G(ICC,1,2)-G(INK,1,2))/G(ICC,1,2)
      TEST2=(G(ICC,NAPE,2)-G(INK,NAPE,2))/G(ICC,NAPE,2)
      TEST3=(F(ICC,NAPE,2)-F(INK,NAPE,2))/F(ICC,NAPE,2)
      TEST4=(C(ICC,NAPE,2)-C(INK,NAPE,2))/C(ICC,NAPE,2)
      LOGIC=ABS(TEST2).GT.1.E-4.OR.ABS(TEST3).GT.1.E-4.OR.ABS(TEST4).GT.1.E-5
      IF(ABS(TEST1).GT.1.E-4)GO TO 300
      IF(LOGIC)GO TO 300
      IL=IL+1
      IF(IL.NE.2)GO TO 4
  280 NX=NXX
      IF(IT.GT.NFG)GO TO 299
      IF((NX+1).NE.NXG(IT))GO TO 299
      CALL STORE
      IT=IT+1
  299 CONTINUE
      CALL OUTPUT
      IF(NX.LE.NXT)GO TO 20
      WRITE (LP,7000)
      CALL EXIT
  300 CONTINUE
      IL=0
      GO TO 40
  400 WRITE (LP,950)NX,X(NX),INT,IT,G(INT,1,2),DELG(1)
      DO9999 I=1,NP
 9999 WRITE(LP,9998)I,ETA(I),F(INT,I,2),G(INT,I,2),C(INT,I,2),
     1               DELG(I),DELC(I)
      WRITE(LP,900)
      STOP 99
C     ------------------------------------
  901 FORMAT(5X,11H*** PERIOD ,I3,4H ***)
  800 FORMAT(////20X,54HNUMERICAL SOLUTION OF PULSATING TURBULENT 2-D FRE
     1E JET////)
  900 FORMAT(///10X,25HITERATIONS EXCEEDED ITMAX)
  950 FORMAT(/5H NX= ,I4,2X,7H X(NX)=,F8.4,2X,4HINT=,I3,2X,4HIT= ,I3,2X,
     1       8HF(NX)=,E12.5,2X,6HDELC=,E12.5)
 7000 FORMAT(///20X,27H*** END OF CALCULATIONS ***)
 9998 FORMAT(I5,6(2X,E14.7))
      END

      SUBROUTINE INPUT
      COMMON/BLC/LC,LP
      COMMON/BL1/NP,NPN(51),NX,NXT,NT,DETA(100),A(100),ETA(100),IFREQ,
     1           INC
      COMMON/BL2/X(100)
      COMMON/BL3/DELC(100),F(51,100,2),C(51,100,2),G(51,100,2)
      COMMON/BL4/INT,INT1,INTP,NPNSX,NPNIN
      COMMON/BL5/C1,C2,C3,C4,C5
      COMMON/BL6/NC(8),DETAS(8),ETAF,NPC(8)
      COMMON/BL7/A,A4,DT,CP,Y1,EPS,OMG
      COMMON/BL13/NT1
      COMMON/BL20/CONST1,CONST2
      COMMON/BL30/ARC,ARC2
      COMMON/BL31/IN,START
      COMMON/BL32/TAU(51,100,2),PRCD
      COMMON/BL33/IPS,IIA
      COMMON/BL35/NTS1NS
      COMMON/BL50/SIGN(51,2)
      COMMON/BL200/FC(5),IG,NMG,PTCR,NXG(5)
      COMMON/BL300/NPRINT
      COMMON/BL800/FLAG
      COMMON/BL900/NNN
```

```
      DIMENSION NXTT(8),DELX(7)
      INTEGER START
C     ------------------------------------
C**** THIS READS IN THE INPUT DATA
C
      READ(LC,1[illegible])NAG,([illegible](I),I=1,NAG),[illegible]
      READ(LC,1[illegible])(NA[illegible](I),[illegible])
      READ(LC,1[illegible])[illegible],NTRANS,NPRINT
      READ(LC,2)CONST1,[illegible]
      READ(LC,[illegible])[illegible],NT,INC
      READ(LC,1[illegible])[illegible]
      NXTT=NXT+1
      READ(LC,1[illegible])(NXTT(I),I=2,NXTT)
      NXTT(1)=1
      NXT=NXTT(NXTT)
      READ(LC,11[illegible])(DELX(I),I=1,NXT),X(1)
      READ(LC,11[illegible])([illegible](I),I=1,[illegible])
      READ(LC,11[illegible])(V[illegible](I),I=1,7),[illegible]
      READ(LC,11[illegible])[illegible],2,[illegible],4,[illegible],P[illegible]
      READ(LC,11[illegible])[illegible]
      WRITE(LP,1[illegible])[illegible],([illegible](I),I=1,[illegible]),[illegible]
      WRITE(LP,14[illegible])(NX[illegible](I),I=1,[illegible])
      WRITE(LP,1[illegible])[illegible],LN,[illegible],NTRANS,NPRINT
      WRITE(LP,2)CONST1,[illegible]
      WRITE(LP,[illegible])[illegible],INC,N[illegible],NPC2,IF[illegible],IF3,IF4,NXI
      WRITE(LP,1[illegible])(NXTT(I),I=1,NXTT)
      WRITE(LP,12[illegible])(DELX(I),I=1,NXT),X(1)
      WRITE(LP,1[illegible])([illegible](I),I=1,[illegible])
      WRITE(LP,12CC)(V[illegible](I),I=1,7),[illegible]
      WRITE(LP,1[illegible])[illegible],[illegible],4,[illegible],P[illegible]
      WRITE(LP,115C)[illegible]
      NT1=NT-1
      DT=6.2831853/(FMC*FLOAT(NT1))
C**** TO CALCULATE REFERENCE SINE/COSINE SIGNAL
      DO 8[illegible] I=1,NT1
      CM=FLOAT(I-1)*FMC*DT
      SIGN(I,1)=SIN(CM)
  8[illegible] SIGN(I,2)=COS(CM)
      DO 9CC I=1,NXT
  9CC CALL CALX(DELX(I),NXTT(I),NXTT(I+1))
      RETURN
C     ------------------------------------
    1 FORMAT([illegible])
    2 FORMAT([illegible]F1[illegible].4)
  5C[illegible] FORMAT(4F1C.4,2I5)
  51[illegible] FORMAT(4[illegible]1[illegible].5,8I5)
 1C2C FORMAT(4[illegible])
 1C5C FORMAT(1[illegible])
 1[illegible] FORMAT([illegible])
 115C FORMAT([illegible]F1[illegible].4)
 12CC FORMAT([illegible]14.7)
 1[illegible] FORMAT(5I5)
 141C FORMAT(5X,[illegible]I1C)
 16[illegible] FORMAT(I5,6F1[illegible].6)
 17CC FORMAT(5X,I5,6(2X,F10.6))
      END

      SUBROUTINE CALX(DELX,NXTI,NXTF)
      COMMON/BL2/X(1CC)
      DO 9CC I=NXTI,NXTF
  9CC X(I+1)=X(I)+DELX
      RETURN
      END

      SUBROUTINE COEFF
      COMMON/BL[illegible]/LC,LP
      COMMON/BL1/NF,FF[illegible](51),NX,NXT,NT,D[illegible](1[illegible]),A(1[illegible]),[illegible](1CC),I[illegible],
     1 I[illegible]
      COMMON/BL2/X(1[illegible])
      COMMON/BL3/[illegible](1[illegible]),F(51,1[illegible],2),C(51,1[illegible],2),C(51,1[illegible],2)
      COMMON/BL4/[illegible],MPMAX,NPMIN
      COMMON/BL7/[illegible],EPS,CMG
      COMMON/BL[illegible]/S1(1C[illegible]),S2(1[illegible]),S3(1CC),S4(1CC),S5(1[illegible]),S6(1CC)
```

```
      COMMON/BLE/R1(100),R2(100),R3(100)
      COMMON/BLE2/TAU(51,100,2)
C
C**** THIS EVALUATES THE COEFFICIENTS OF THE LINEARIZED SYSTEM RESULTING
C**** FROM NEWTON'S METHOD.
C
      INT1=INT-1
      IF(INT.EQ.1)INT1=NT-1
      RM=0.5*(1./SQRT(X(NX)+XC)+1./SQRT(X(NX-1)+XC))
      ALP=0.5*(X(NX)+X(NX-1)+2.*XC)/(X(NX)-X(NX-1))
      ALP1=2.*((X(NX)+XC)**1.5+(X(NX-1)+XC)**1.5)/(DT*AA*BB)
      ALP2=1.-ALP
      ALP3=1.+ALP
C**** PRESENT STATION
      DO 2500 J=2,NP
      GSB=0.5*(G(INT,J,2)**2+G(INT,J-1,2)**2)
      FGB=0.5*(F(INT,J,2)*G(INT,J,2)+F(INT,J-1,2)*G(INT,J-1,2))
      FB=0.5*(F(INT,J,2)+F(INT,J-1,2))
      GB=0.5*(G(INT,J,2)+G(INT,J-1,2))
      QB=0.5*(Q(INT,J,2)+Q(INT,J-1,2))
      GTJH=0.5*(G(INT,J,1)+G(INT1,J,2)+G(INT1,J,1)+G(INT,J-1,1)+
     1             G(INT1,J-1,2)+G(INT1,J-1,1))
      GIN1H=0.5*(G(INT1,J,2)+G(INT1,J-1,2))
      BGJ1H=TAU(INT,J-1,2)*G(INT,J-1,2)
      BGJB=TAU(INT,J,2)*G(INT,J,2)
C**** PREVIOUS STATION
      GRI1=0.25*(G(INT,J,1)+G(INT,J-1,1)+G(INT1,J,1)+G(INT1,J-1,1))
      FIN1H=0.5*(F(INT1,J,2)+F(INT1,J-1,2))
      FRI1=0.25*(F(INT,J,1)+F(INT,J-1,1)+F(INT1,J,1)+F(INT1,J-1,1))
      GTJ1=Q(INT1,J-1,2)+Q(INT1,J-1,1)+Q(INT,J-1,1)
      GTJ=Q(INT1,J,2)+Q(INT1,J,1)+Q(INT,J,1)
      FGTJH=0.5*(F(INT1,J,2)*G(INT1,J,2)+F(INT1,J,1)*G(INT1,J,1)+
     1             F(INT,J,1)*G(INT,J,1)+F(INT1,J-1,2)*G(INT1,J-1,2)+
     2             F(INT1,J-1,1)*G(INT1,J-1,1)+F(INT,J-1,1)*G(INT,J-1,1))
      GSTJH=0.5*(G(INT1,J,2)**2+G(INT1,J,1)**2+G(INT,J,1)**2+
     1             G(INT1,J-1,2)**2+G(INT1,J-1,1)**2+G(INT,J-1,1)**2)
      GTJH=0.5*(GTJ1+GTJ)
      GNI1H=0.5*(G(INT,J,1)+G(INT,J-1,1))
      GRN1=0.25*(G(INT1,J,2)+G(INT1,J-1,2)+G(INT1,J,1)+G(INT1,J-1,1))
      BGTJ=TAU(INT,J,1)*G(INT,J,1)+TAU(INT1,J,2)*G(INT1,J,2)+
     1       TAU(INT1,J,1)*G(INT1,J,1)
      BGTJ1=TAU(INT,J-1,1)*G(INT,J-1,1)+TAU(INT1,J-1,2)*G(INT1,J-1,2)+
     1       TAU(INT1,J-1,1)*G(INT1,J-1,1)
C**** COEFFICIENTS OF THE DIFFERENCED MOMENTUM EQUATION
      DET=0.5*DETA(J-1)
      BETA=GTJH+GIN1H-2.*GRI1
      BETA1=FIN1H-2.*FRI1
      BETA2=GIN1H-2.*GRI1
      BETA3=GNI1H-2.*GRN1
      S1(J)=RM*TAU(INT,J,2)+DET*(F(INT,J,2)+ALP*FB+ALP*BETA1)
      S2(J)=-RM*TAU(INT,J-1,2)+DET*(F(INT,J-1,2)+ALP*FB+ALP*BETA1)
      S3(J)=DET*(G(INT,J,2)+ALP*GB+ALP*GTJH)
      S4(J)=DET*(G(INT,J-1,2)+ALP*GB+ALP*GTJH)
      S5(J)=DETA(J-1)*(G(INT,J,2)-ALP*GB-0.5*ALP*BETA-0.5*ALP1)
      S6(J)=DETA(J-1)*(G(INT,J-1,2)-ALP*GB-0.5*ALP*BETA-0.5*ALP1)
C**** EVALUATIONS OF RJ
      TRP=RM*(BGTJ-BGTJ1)+DETA(J-1)*(-GSTJH-FGTJH+ALP*(BETA2*GTJH
     1      -BETA1*GTJH)+ALP1*BETA3)
      R1(J)=F(INT,J-1,2)-F(INT,J,2)+DETA(J-1)*GB
      R2(J)=TRP+RM*(BGJ1H-BGJB)-DETA(J-1)*(GSB+FGB-ALP*(0.5*(GSB+
     1         G(INT,J,2)*G(INT,J-1,2))+BETA*GB-FB*GB-BETA1*GB-
     2         GTJH*FB)-ALP1*GB)
      R3(J-1)=G(INT,J-1,2)-G(INT,J,2)+DETA(J-1)*GB
      IF(J.NE.55)GO TO 2500
      WRITE(LP,4000)G(INT,J,1),G(INT,J-1,1),G(INT1,J,1),G(INT1,J-1,1),J
      WRITE(LP,1999)GSB,FGB,FB,GB,QB,GTJH,GIN1H,GRI1,FIN1H,FRI1,GTJ1,
     1                 GTJ,FGTJH,GSTJH,GEJH,GNI1H,GRN1,R1(J),R2(J),R3(J-1),
     2                 DETA(J-1)
      WRITE(LP,2000)S1(J),S2(J),S3(J),S4(J),S5(J),S6(J),TRP
      WRITE(LP,3000)G(INT,J-1,2),G(INT,J,2),G(INT1,J-1,2),G(INT1,J,2)
      DO 1000 I=1,NP
 1000 WRITE(LP,1001)I,ETA(I),F(INT,I,2),G(INT,I,2),Q(INT,I,2)
 1001 FORMAT(I5,4(2X,E14.7))
 2500 CONTINUE
```

```
      B1(1)=0.
      B2(1)=1.
      B3(NP)=0.
      RETURN
C     -------------------------------------
 1990 FORMAT(6H0C    ,5(1X, E12.5))
 2000 FORMAT(5(2X,E12.5))
 4000 FORMAT(4H CP ,4(2X,E12.5),I5)
      END

      SUBROUTINE CELL
      DIMENSION W1(100),W2(100),W3(100),A11(100),A12(100),A13(100)
      DIMENSION A21(100),A22(100),A23(100),C11(100),C12(100),C13(100)
      DIMENSION C21(100),C22(100),C23(100),DELF(100)
      COMMON/BL /NP,NI
      COMMON/BL1/  (11),DX,DT,RI,DETA(100),X(100),ETA(100),TEPTQ,
     1
      COMMON/BL2/Y(100)
      COMMON/BL3/ (100), (51,100,2),C(51,100,2),G(51,100,2)
      COMMON/BL4/  ,  ,  ,  ,NPP1
      COMMON/BL5/ (100), (100), (100),S4(100),S5(100),S6(100)
      COMMON/BL6/ (100), (100), (100)
      COMMON/BL7/
      COMMON/BL8/ LG(100)
C     -------------------------------------
C**** THIS SOLVES THE LINEAR(IZED) SYSTEM BY THE BLOCK ELIMINATION METHOD.
C
C**** W ELEMENTS FOR J=1
      W1(1)=R1(1)
      W2(1)=R2(1)
      W3(1)=R3(1)
C**** ALPHA ELEMENTS FOR J=1
      A11(1)=1.
      A12(1)=0.
      A13(1)=0.
      A21(1)=0.
      A22(1)=0.
      A23(1)=1.
C**** GAMMA ELEMENTS FOR J=2
      G11(2)=-1.
      G12(2)=A(2)*2
      G13(2)=S(2)
      G21(2)=S4(2)
      G22(2)=S2(2)-S(2)*S6(2)
      G23(2)=-S6(2)
C**** FORWARD SWEEP
      DO 3050 J=2,NP
      IF(J.EQ.2)GO TO 3000
      DDC=A13(J-1)*A21(J-1)-A23(J-1)*A11(J-1)-S(J)*(A12(J-1)*A21(J-1)-
     1A22(J-1)*A11(J-1))
      DD1=A22(J-1)*A(J)-A21(J-1)
      G11(J)=(A23(J-1)+A(J)*(S(J)*A21(J-1)-A22(J-1)))/DDC
      G12(J)=-(A(J)*A(J)+G11(J)*(A12(J-1)*A(J)-A13(J-1)))/DD1
      G13(J)=(G11(J)*A13(J-1)+G12(J)*A23(J-1))/A(J)
      G21(J)=(S2(J)*A21(J-1)-S4(J)*A23(J-1)+S(J)*(S4(J)*A22(J-1)-S6(J)*
     1A21(J-1)))/DDC
      G22(J)=(-S2(J)*S6(J)*A(J)-G21(J)*(A(J)*A12(J-1)-A13(J-1)))/DD1
      G23(J)=G21(J)*A13(J-1)+G22(J)*A22(J-1)-S6(J)
 3000 A11(J)=1.
      A12(J)=-A(J)-G13(J)
      A13(J)=S(J)*G13(J)
      A21(J)=S2(J)
      A22(J)=S5(J)-G23(J)
      A23(J)=S1(J)+S(J)*G23(J)
      W1(J)=R1(J)-G11(J)*W1(J-1)-G12(J)*W2(J-1)-G13(J)*W3(J-1)
      W2(J)=R2(J)-G21(J)*W1(J-1)-G22(J)*W2(J-1)-G23(J)*W3(J-1)
      W3(J)=R3(J)
 3050 CONTINUE
C**** BACKWARD SWEEP
      DELG(NP)=W3(NP)
      E1=W1(NP)-A13(NP)*DELG(NP)
      E2=W2(NP)-A23(NP)*DELG(NP)
      DELG(NP)=(E2*A11(NP)-E1*A21(NP))/(A22(NP)*A11(NP)-A12(NP)*A21(NP))
      DELF(NP)=(E1-A12(NP)*DELG(NP))/A11(NP)
```

```
      J=NP
 3100 J=J-1
      B=W2(J)-DELUC(J+1)*A(J+1)*DELUC(J+1)
      DEP=A21(J)*A12(J)*A(J+1)-A21(J)*A13(J)-A(J+1)*A22(J)*A11(J)+A23(J)
     1*A11(J)
      DELUC(J)=(A11(J)*(W2(J)+B3*A22(J))-A21(J)*W1(J)-B3*A21(J)*A12(J))/
     1DEP
      DELG(J)=-C(J+1)*DELUC(J)-B3
      DELF(J)=(W1(J)-A12(J)*DELG(J)-A13(J)-DELUC(J))/A11(J)
      IF(J.GT.1)GO TO 3100
      IF(J.EQ.99)WRITE(LP,3200)IT,C(INT,1,2),DELUC(1)
      DO 3150 J=1,NP
      F(INT,J,2)=F(INT,J,2)+DELF(J)
      C(INT,J,2)=C(INT,J,2)+DELUC(J)
      G(INT,J,2)=G(INT,J,2)+DELG(J)
 3150 CONTINUE
      G(INT,1,2)=0.
      RETURN
C     ---------------------------------
 3200 FORMAT(5X,'IT =',I6,5X,'UMAX = ',E14.6,5X,'DELUC = ',E14.6)
      END

      SUBROUTINE OUTAN
      COMMON/BL0/LC,LP
      COMMON/BL1/NP,NPA(51),NX,NXT,NT,DETA(100),A(100),ETA(100),IPRED,
     1            INC
      COMMON/BL2/DELUC(100),F(51,100,2),C(51,100,2),G(51,100,2)
      COMMON/BL7/PP,EB,ST,CC,XO,EPS,CMG
      COMMON/BL13/NT1
      COMMON/BL2/X(100)
      COMMON/BL16/ANC,AGO
C     ---------------------------------
C**** THIS EVALUATES THE MEAN MOMENTUM FLUX AND ENTRAINMENT
C
      EXP1(U1,U2,U3,DJ1,DJ)=(DJ1+DJ)*((2.*DJ1-DJ)*DJ*U1+
     1((DJ1+DJ)**2)*U2+(2.*DJ-DJ1)*DJ1*U3)/(6.*DJ1*DJ)
      EXP2(U1,U2,DJ1)=0.5*DJ1*(U1+U2)
      AM=0.
      AMF=0.
      DO 5030 I=1,NT1
      J=2
      JW=NPA(I)
 5000 U1=G(I,J-1,2)*C(I,J-1,2)
      U2=G(I,J,2)*C(I,J,2)
      U3=G(I,J+1,2)*C(I,J+1,2)
      DJ1=DETA(J-1)
      DJ=DETA(J)
      AM=AM+EXP1(U1,U2,U3,DJ1,DJ)
      AMF=AMF+EXP1(G(I,J-1,2),G(I,J,2),G(I,J+1,2),DJ1,DJ)
      IF((J+1).EQ.JW)GO TO 5020
      J=J+2
      DJ1=DETA(J-1)
      IF((J+1).LT.JW)GO TO 5000
      U1=G(I,J-1,2)*C(I,J-1,2)
      U2=G(I,J,2)*C(I,J,2)
      AM=AM+EXP2(U1,U2,DJ1)
      AMF=AMF+EXP2(G(I,J-1,2),G(I,J,2),DJ1)
 5020 CONTINUE
 5030 AMF=SQRT(X(NX)+XO)*AMF
      AM=AM/FLOAT(NT1)
      AMF=AMF/FLOAT(NT1)
      IF(NX.NE.1)GO TO 5050
      ANC=AM
      AGO=AMF
 5050 CONTINUE
      AMC=100.*AM/ANC
      AGO=100.*AMF/AGO
      WRITE(LP,5100)AM,AMC,AMF,AGO
      RETURN
C     ---------------------------------
 5100 FORMAT(10X,13HAVE MOM AM = ,E15.7,7X,7HM/MO = ,F6.2,9H PER CENT/
     1       10X,15HAVE MASS AMF = ,E15.7,5X,7HQ/QO = ,F6.2,9H PER CENT)
      END
```

```
      SUBROUTINE OSCT(II,YY)
      COMMON/BL3/DELC(100),F(51,100,2),C(51,100,2),Q(51,100,2)
      COMMON/BL13/NT)
C     ----------------------------------
C**** THIS EVALUATES A LOCAL MAXIMUM BY QUADRATIC INTERPOLATION
C**** THROUGH THREE POINTS
C
      IC2=II
      IC1=IC2-1
      IC3=IC2+1
      IF(IC2.LE.1)IC1=IC3
      IF(IC2.EQ.NT))IC3=IC1
      RA1=IC1
      RA2=RA1+1
      AA=0.5*(C(IC1,1,2)+C(IC3,1,2)-2.*C(IC2,1,2))
      BB=C(IC2,1,2)-C(IC1,1,2)-AA*(RA1+RA2)
      CC=C(IC1,1,2)-AA*RA1*RA1-BB*RA1
      XX=-0.5*BB/AA
      YY=AA*XX*XX+BB*XX+CC
      RETURN
C     ----------------------------------
      END

      SUBROUTINE INITC
      COMMON/BL1/NP,NPA(51),NX,NXT,NT,DETA(100),X(100),ETA(100),IFRIG,
     1          INC
      COMMON/BL2/X(100)
      COMMON/BL3/DELC(100),F(51,100,2),C(51,100,2),Q(51,100,2)
      COMMON/BL4/INT,ICT,ITR,IMAX,NPAIN
      COMMON/BL12/NPAC(51)
      COMMON/BL11/LN,ISTART
      COMMON/BL22/TF(51,100,2)
      INTEGER START
      DATA ISTART/0/
C     ------------------------------------
C**** TO READ INITIAL CONDITIONS IN (X,ETA) PLANE FROM DATA FILES
C
      ISTART=ISTART+1
      ICOMP=START+1
      IF(ISTART.GT.1)GO TO 6000
      DO 6900 I=1,100
      READ(LN)NX
      IF((NX+1).EQ.ICOMP)GO TO 6900
 6800 CONTINUE
 6900 READ(LN)NX,NP,X(NX),(F(INT,J,2),J=1,NP),(C(INT,J,2),J=1,NP),
     1              (Q(INT,J,2),J=1,NP),(INC(INT,J,2),J=1,NP)
      NPA(INT)=NP
      NPI=NPAC(INT)
      IF(NPI.GE.NP)RETURN
      NP1=NPI+1
      DO 4000 J=NP1,NP
      CC=ETA(J)-ETA(NPI)
      F(INT,J,1)=F(INT,NPI,1)+Q(INT,NPI,1)*CC
      Q(INT,J,1)=Q(INT,NPI,1)
      C(INT,J,1)=0.
 4000 INC(INT,J,1)=0.
      RETURN
C     ----------------------------------
      END

      SUBROUTINE GRID
      COMMON/BL0/LC,LP
      COMMON/BL1/NP,NPA(51),NX,NXT,NT,DETA(100),X(100),ETA(100),IFRIG,
     1          INC
      COMMON/BL2/X(100)
      COMMON/BL4/INT,ICT,ITR,IMAX,NPAIN
      COMMON/BL6/VC(8),DETAC(8), IAF,NFC(8)
      COMMON/BL12/NPAC(51)
C     ----------------------------------
C**** WITH SUBROUTINE EVAL, THIS SETS UP A NETWORK OF VARIABLE GRID SIZE
C
      ETA(1)=0.
      NP=2
      NPC(1)=VC(1)/DETAC(1)+1.
```

```
      DO 100 I=1,7
      IF(I.EQ.1)GO TO 50
      NPC(I)=NPC(I-1)+IFIX((VC(I)-VC(I-1))/DETAC(I))
   50 IF(VC(I).EQ. TAC)L=I
      IF(NPC(I).GT.100)NPC(I)=100
      CALL  VAL(NPP,NPC(I),VC(I),D TAC(I))
      NPP=NPC(I)+1
      IF(NPP.GT.100)GO TO 1700
  100 CONTINUE
      VC(8)=VC(7)+DETAC(8)*FLOAT(100-NPC(7))
      CALL  VAL(NPP,100,VC(8),D TAC(8))
 1700 CONTINUE
      NP=NPC(L)
      DO 150 I=1,NT
      NPA(I)=NP
  150 NPP(I)=NP
      IF(NPP.GT.100)WRITE(LP,1800)ETA(100)
      RETURN
C     ----------------------------------
 1800 FORMAT(1OX,27H*** NP EXCEEDED 100 ***,5X,11HETA(100) = ,F10.6)
      END

      SUBROUTINE REGRID
      COMMON/BLC/LC,LP
      COMMON/BL1/NF,NPA(51),NX,NXT,NT,DETA(100),A(100),ETA(100),IERRO,
     1           INC
      COMMON/BL3/D EG(100),F(51,100,2),G(51,100,2),Q(51,100),2)
      COMMON/BL4/INT,INT1,INTP,NEMAX,NEMIN
      COMMON/BL6/VC(8),DETAC(8),ETAL,NPC(8)
      COMMON/BL13/NT1
      COMMON/BL23/TNL(51,100,2),EDGE
      COMMON/BL200/AC(5),IT,NPG,ETGR,NXG(5)
C     ----------------------------------
C**** TO RE-SET THE GRID SYSTEM BY DROPPING SPECIFIED POINTS
C
      WRITE(LP,110)
      DO 50 I=1,100
      IF(ETA(I).NE.ETGR)GO TO 50
      IND=I
      GO TO 60
   50 CONTINUE
   60 CONTINUE
      INT=IND+1
      DO 30 I=1,8
      IF(VC(I).LT.EG(ID))GO TO 30
      IVC=I-1
      IF(IC.EQ.NPC)IVC=I
      GO TO 40
   30 CONTINUE
   40 CONTINUE
      ETGR=VC(IVC)
      DO 100 K=1,NT1
      NP=NPA(K)
      INJ=ICC
      DO 70 I=INT,NP
      IC=IND+2*(I-IND)
      IF(ETA(IC).LT.VC(IVC))GO TO 65
      IF(ETA(IC-2).NE.VC(IVC))GO TO 68
      INJ=I-1
      ICC=IC-2
      GO TO 80
   68 INJ=I
      ICC=IC-1
      GO TO 80
   65 CONTINUE
      F(K,I,2)=F(K,IC,2)
      G(K,I,2)=G(K,IC,2)
      Q(K,I,2)=Q(K,IC,2)
      TNL(K,I,2)=TNL(K,IC,2)
   70 CONTINUE
   80 IF(ETA(NP).GT.VC(IVC))NPA(K)=INJ+NP-ICC
      IF(ETA(NP).LT.VC(IVC))NPA(K)=NP-(NP-IND+1)/2
      IF(ETA(NP).EQ.VC(IVC))NPA(K)=INJ
      NP=NPA(K)
```

```
      IF(NCAT .GT.NB)NMIN=IC-1
      IF(NB.[illegible].IN)GO TO 1[illegible]
      DO 90 I=IN,NB
      IC=IC +I-INJ
      F(K,I,2)=F(K,IC,2)
      C(K,I,2)=C(K,IC,2)
      G(K,I,2)=G(K,IC,2)
      TAU(K,I,2)=TAU(K,IC,2)
   90 CONTINUE
  100 CONTINUE
      NPP=INJ
      IC=2*INJ-INC
      IF(IC.GT.1[illegible])NPP=(1[illegible]+INC)/2
      DO 200 I=NPP,NFF
      IC=INJ+2*(I-INC)
      ETA(I)=ETA(IC)
      IF(ETA(I).LE.VC(IVC))INJ=I
      DETA(I-1)=ETA(I)-ETA(I-1)
  200 Z(I)=0.5*DETA(I-1)
      NPC(IVC)=INJ
      NPP=INJ+1
      IVC1=IVC+1
      IF(IVC1.GT.7)GO TO 105
      DO 1 0 I=IVC1,7
      NPC(I)=NPC(I-1)+IFIX((VC(I)-ETA(NPP-1))/DETAC(I)+0.5)
      IF(NPC(I).GT.100)NPC(I)=100
      CALL  VAL(NPP,NPC(I),VC(I),DETAC(I))
      NPP=NPC(I)+1
      IF(NPP.GT.100)RETURN
  1   CONTINUE
  105 CONTINUE
      VC(8)=VC(7)+DETAC(8)*FLOAT(100-NPC(7))
      CALL  VAL(NPP,100,VC(8),DETAC(8))
      DO 120 I=1,NFF
  120 WRITE(LP,130)I,ETA(I),DETA(I),Z(I)
      RETURN
C     ------------------------------------
  110 FORMAT(10X,35H***** GRID RESTRUCTURED *****)
  130 FORMAT(I5,3(2X,F8.5))
      END

      SUBROUTINE EVAL(NPT,NPF,P,DETAS)
      COMMON/BL1/NP,NPA(51),DX,DXT,NT,DETA(100),Z(100),ETA(100),TERTC,
     1            INC
      COMMON/BL2/X(100)
C     ------------------------------------
      DO 1750 J=NPT,NPF
      DETA(J-1)=DETAS
      Z(J)=0.5*DETA(J-1)
      ETA(J)=ETA(J-1)+DETA(J-1)
 1750 CONTINUE
      IF(NPF.NE.100)RETURN
      DPR=(ETA(NPF)-P)/P
      IF(ABS(DPR).GT.0.001)GO TO 1900
      ETA(NPF)=P
      RETURN
 1900 NPF=NPF+1
      ETA(NPF)=P
      DETA(NPF-1)=ETA(NPF)-ETA(NPF-1)
      Z(NPF)=0.5*DETA(NPF-1)
      RETURN
C     ------------------------------------
      END

      SUBROUTINE GROWTH
      COMMON/BL0/LC,LP
      COMMON/BL1/NP,NPA(51),DX,DXT,NT,DETA(100),Z(100),ETA(100),TERTC,
     1            INC
      COMMON/BL2/X(100)
      COMMON/BL3/[illegible](100),F(51,100,2),E(51,100,2),G(51,100,2)
      COMMON/BL4/[illegible],[illegible],INTR,[illegible],NPNIS
      COMMON/BL6/VC(8),DETAC(8),IVC,NPC(8)
      COMMON/BL10/[illegible](51)
      COMMON/BL30/[illegible],NPC2
```

```
      COMMON/BL32/TNU(51,100,2)
C     ----------------------------------
C**** THIS ALLOWS FOR THE SPREADING OF THE JET.
C
      NPC=NP
      NP=NP+NPC
      IF(NP.GT.100)STOP 50
      IF(ETA(NP).GT.VC(7))NP=NP-NPC+NPC2
C**** DEFINITION OF PROFILES FOR NEW NP
      INTT=INT-1
      IF(INTT.EQ.0)INTT=NT-1
      IF(NPS(INTT).GT.NP)INTT=INT
      K=INTT
      IL=1
 4000 CONTINUE
      DO4060 I=1,2
      NP1=NPS(K)+1
      IF(I.EQ.2)NP1=NPS(K)+1
      IF(NP1.GT.NP)GO TO 4060
      DO4050 J=NP1,NP
      CC=ETA(J)-ETA(NPC)
      IF(I.EQ.1)TNU(K,J,1)=0.
      IF(I.EQ.2.AND.K.EQ.INT)TNU(K,J,2)=TNU(K,NPC,2)
      IF(I.EQ.2.AND.K.NE.INT)TNU(K,J,2)=0.
      F(K,J,I)=G(K,NPC,I)*CC+F(K,NPC,I)
      G(K,J,I)=G(K,NPC,I)
 4050 Q(K,J,I)=0.
 4060 CONTINUE
      IF(IL.EQ.2)GO TO 5000
      IF(INTT.EQ.INT)GO TO 5000
      K=INT
      IL=IL+1
      GO TO 4000
 5000 CONTINUE
      NPS(INT)=NP
      NPMAX=NP
      RETURN
C     ----------------------------------
      END

      SUBROUTINE IVPL
      COMMON/BLC/LC,LP
      COMMON/BL1/NF,NPS(51),NX,NXT,NT,DETA(100),A(100),ETA(100),IFREQ,
     1           TNC
      COMMON/BL2/X(100)
      COMMON/BL3/DELC(100),F(51,100,2),G(51,100,2),Q(51,100,2)
      COMMON/BL4/VC(8),OTAC(8),ITAB
      COMMON/BL7/AA,BB,CT,CD,X3,EPS,OMG
      COMMON/BL12/NT1
      COMMON/BL31/NAM,NAM1,NAM2,NAM3,LN,NR,LN1,NR1,LN2,NR2,LN3,NR3,START
      COMMON/BL32/TNU(51,100,2)
      COMMON/BL33/IFR,IFA
      COMMON/BL40/MODE,MODE1,MODE2
      COMMON/BL70/ISIZE1,ISIZE2,ISIZE3
      COMMON/BL100/NXX
      COMMON/BL500/ETAC
      INTEGER START
C     ----------------------------------
C**** THIS GENERATES THE TIME VARYING VELOCITY PROFILE
C
      DO190 J=1,NP
      IF(ETA(J).LE.ETAC)GO TO 180
      JJ=J-1
      GO TO 190
  180 CONTINUE
  190 IT=JJ+1
      CC1=ETA(IT)-ETA(JJ)
      DO200 I=1,NT
      OM=FLOAT(I-1)*6.2831853/FLOAT(NT1)
      P=1.+EPS*SIN(OM)
      DO200 J=1,NX
      G(I,J,2)=CC1*P
      F(I,J,2)=ETA(J)*G(I,J,2)
  200 Q(I,J,2)=0.
```

```
      DO 1000 J=II,NP
      DC=ETA(J)-ETA(JJ)
      ETAD=DC/DC1
      F(I,J,2)=F(I,JJ,2)+[illegible]*(1.-[illegible]+0.5*ETAD*ETAD*ETAD)
      C(I,J,2)=CC*(1.-ETAD)*(1.-[illegible])*(1.+2.*ETAD)
 1000 G(I,J,2)=-6.*CCK TAC*(1.-[illegible])/[illegible]
 2000 CONTINUE
      RETURN
C     --------------------------------
      END

      SUBROUTINE EDDY
      COMMON/BL1/NP,NPS(51),NX,NXT,[illegible],ETA(100),[illegible](100),ETA(100),IERRG,
     1            INC
      COMMON/BL2/X(100)
      COMMON/BL3/DELC(100),F(51,100,2),[illegible](51,100,2),G(51,100,2)
      COMMON/BL4/INT,INT1,INT2,[illegible]
      COMMON/BL7/[illegible]
      COMMON/BL[illegible]/CONST1,CONST2
      COMMON/BL[illegible]/TNL(51,100,2),[illegible]
      COMMON/BL[illegible]/[illegible]
      COMMON/BL[illegible]/[illegible]
C     --------------------------------
C**** THIS CALCULATES THE EFFECTIVE EDDY VISCOSITY
C
      DO 2000 I=1,NP
      COMP1=C(INT,I,2)/C(INT,1,2)
      COMP2=C(INT,I+1,2)/C(INT,1,2)
      IF(ABS(COMP1).LT.0.50.AND.ABS(COMP2).GT.0.50)GO TO 2000
      II=I
      GO TO 2010
 2000 CONTINUE
 2010 WIDTH1=ETA(II)+(COMP1-0.5)*[illegible]TA(II)/(COMP1-COMP2)
      RNIX=X(INX)+XC
      CONST=CONST1
      IF(X(INX).GE.XTRANS)CONST=CONST2
      TCONST=CONST*RENG(INT,1,2)*SQRT(RNIX)*WIDTH1*PROD
      DO 2050 I=1,NP
 2050 TNL(INT,I,2)=TCONST
      RETURN
C     --------------------------------
      END

      SUBROUTINE OUTPUT
      DIMENSION GNEF(100)
      DIMENSION AVE(100,3)
      DIMENSION CC(51),[illegible]LE(51),C[illegible](51,100),GHA(100)
      COMMON/BL0/IC,LP
      COMMON/BL1/NP,NPS(51),NX,NXT,NT,DETA(100),A(100),ETA(100),IERRG,
     1            INC
      COMMON/BL2/X(100)
      COMMON/BL3/DELC(100),F(51,100,2),C(51,100,2),G(51,100,2)
      COMMON/BL4/INT,INT1,INT2,CMAX,NCMIN
      COMMON/BL5/[illegible]
      COMMON/BL7/[illegible],XN,RES,IMG
      COMMON/BL12/NPSC(51)
      COMMON/BL13/NT
      COMMON/BL[illegible]/NAM,NAM1,NAM2,NAM3,LN,NF,LN1,NP1,LN2,NP2,LN3,NP3,START
      COMMON/BL[illegible]/TNL(51,100,2)
      COMMON/BL[illegible]/TER,IC[illegible]
      COMMON/BL34/IT
      COMMON/BL35/[illegible]SK
      COMMON/BL36/[illegible]
      COMMON/BL[illegible]/STCS(51,2)
      COMMON/BL[illegible]/[illegible]ACC
      COMMON/BL[illegible]/[illegible](5),IC,[illegible],[illegible],NXC(5)
      COMMON/BL[illegible]/NPRINT
      DATA III/1/,III/1/,KKK/1/
C     --------------------------------
C**** THIS OUTPUTS ALL THE REQUIRED QUANTITIES
C
      I[illegible]=I[illegible]-1
      IF([illegible].EQ.[illegible])I[illegible]=1
      IF(NX.LT.NPRINT)GO TO 2100
```

```
      III=III+1
      JJJ=JJJ+1
 2000 CONTINUE
      AGC=0.
      JW=0
      NPS=NPMIN-3
C**** TO COMPUTE MEAN JET CENTER-LINE G AND NO. OF TRANSVERSE GRID POINT
      DO 6000 I=1,NT
      J1=NPA(I)
      NPAA=J1-1
      DO 5950 J=NPS,NPAA
      IF(ABS(G(I,J,2)/G(I,1,2)).GT.E3.OR.ABS(G(I,J+1,2)).GT.E2)
     1  GO TO 5920
      J1=J+1
      GO TO 5950
 5920 CONTINUE
 5950 CONTINUE
      NPA(I)=J1
      NPAO(I)=J1
      AGC=AGC+G(I,1,2)
      JW=JW+J1
 6000 CONTINUE
      NCW=0
      JW=JW-NPA(NT)
      AGC=AGC-G(NT,1,2)
      AGC=AGC/FLOAT(NT1)
      AJW=FLOAT(JW)/FLOAT(NT1)
      JW=AJW
 6010 CONTINUE
      NPMAX=JW
      NPMIN=JW
      DO 6005 I=1,NT1
      IF(NPA(I).LT.NPMIN)NPMIN=NPA(I)
      IF(NPA(I).GT.NPMAX)NPMAX=NPA(I)
 6005 CONTINUE
      DO 6018 I=1,NT1
      IJ=NPA(I)
      IJ1=IJ+1
      IF(IJ1.GT.NPMAX)GO TO 6018
      DO 6016 J=IJ1,NPMAX
      F(I,J,2)=F(I,IJ,2)
      C(I,J,2)=C(I,IJ,2)
 6016 G(I,J,2)=G(I,IJ,2)
 6018 CONTINUE
C**** TO CALCULATE AMPLITUDE OF OSCILLATION
      IMAX=1
      IMIN=1
      UMAX=G(1,1,2)
      UMIN=G(1,1,2)
      DO 6030 I=2,NT1
      IF(G(I,1,2).LE.UMAX)GO TO 6020
      UMAX=G(I,1,2)
      IMAX=I
      GO TO 6030
 6020 IF(G(I,1,2).GE.UMIN)GO TO 6030
      UMIN=G(I,1,2)
      IMIN=I
 6030 CONTINUE
      CALL OSCI(IMAX,YMAX)
      CALL OSCI(IMIN,YMIN)
      PCT=100.*(YMAX-YMIN)/AGC
C**** TO COMPUTE MEAN PROFILES
      DO 6115 I=1,NT
      NJ=NPA(I)
      IF(JW.LE.NJ)GO TO 6115
      NJ1=NJ+1
      DO 6110 J=NJ1,JW
      F(I,J,2)=F(I,NJ,2)
      C(I,J,2)=C(I,NJ,2)
      G(I,J,2)=G(I,NJ,2)
 6110 CONTINUE
 6115 CONTINUE
      DO 6150 J=1,JW
      AVP(J,1)=F(I,J,2)
```

```
      AVP(J,2)=C(1,J,2)
      AVP(J,3)=C(1,J,3)
      DO 613 I=2,NT
      AVP(J,1)=AVP(J,1)+C(I,J,1)
      AVP(J,2)=AVP(J,2)+C(I,J,2)
  613 AVP(J,3)=AVP(J,3)+C(I,J,3)
      DO 6140 K=1,3
 6140 AVP(J,K)=AVP(J,K)/FLOAT(NT1)
      CNTS(J)=AVP(J,3)/UCP
 6150 CONTINUE
      DO 9200 J=1,JY
      IF(CNTS(J).GT.0.5)GO TO 9200
      JH=J
      GO TO 9222
 9200 CONTINUE
 9222 [illegible]=YT(JH-1)+(YT(JH)-YT(JH-1))*(CNTS(JH-1)-0.5)/(CNTS(JH-1)-CNTS(JH))
      [illegible]
C**** TO CALCULATE PHASE [illegible] BY CROSS CORRELATION
      SUM1=0.
      SUM2=0.
      DO 500 I=1,NT1
      SUM1=SUM1+C(I,1,2)*[illegible](I,1)
  500 SUM2=SUM2+C(I,1,2)*[illegible](I,2)
      PHASE=ATAN2(SUM2,SUM1)*180./3.1415[illegible]
      IF(SUM2.GT.0.)PHASE=PHASE-360.
C**** TO OUTPUT [illegible] QUANTITIES
      KKK=KKK+1
      IF(KKK.NE.[illegible])GO TO 9[illegible]
      KKK=0
      WRITE(LP,6[illegible])NX,X(NX),[illegible],IT,C(INT,1,2),DELS(1)
      WRITE(LP,65[illegible])[illegible],NFRIN
      WRITE(LP,[illegible])[illegible]
      WRITE(LP,[illegible])[illegible]
      WRITE(LP,[illegible])PHASE
      CALL [illegible]
 9[illegible] CONTINUE
C**** TO OUTPUT INSTANTANEOUS VELOCITY PROFILES
      IVAR=1
      IEND=4
      IF(NX.EQ.NPRINT)GO TO 801
      IF((NX+1).EQ.NXG(IPD).OR.(NX+2).EQ.NXG(IPD))GO TO 801
      IF(III.NE.IPR)GO TO [illegible]
      III=0
  801 CONTINUE
C**** TO CALL SMOOTHING ROUTINE
      CALL SMOOTH
      WRITE(LP,[illegible])
      WRITE(LP,[illegible])
  800 CONTINUE
      ITEMP=NPA(IVAR)
      IF(IVAR.GT.[illegible])GO TO 820
      IVAR1=IVAR+1
      DO 820 I=IVAR1,IEND
      IF(NPA(I).GT.ITEMP)ITEMP=NPA(I)
  820 CONTINUE
      DO 850 J=1,ITEMP
      DO 850 I=IVAR,IEND
  850 CN(I,J)=C(I,J,2)/C(I,1,2)
      IF(NX.EQ.NPRINT)GO TO 805
      IF((NX+1).EQ.NXG(IPD).OR.(NX+2).EQ.NXG(IPD))GO TO 805
      IF(III.NE.[illegible])GO TO 9[illegible]
  805 CONTINUE
      DO 880 J=1,ITEMP,INC
  88[illegible] WRITE(LP,[illegible])[illegible]YT(J),(CN(I,J),C(I,J,3),I=IVAR,IEND)
  900 CONTINUE
      IF(IEND.EQ.NT1)GO TO 6195
      IVAR=IEND+1
      IEND=IEND+4
      IF(IEND.GT.NT1)IEND=NT1
      WRITE(LP,[illegible])
      GO TO 800
C**** TO COMPUTE INSTANTANEOUS JET HALF WIDTH AND CENTRE-LINE VELOCITY
 6195 DO [illegible] I=1,NT1
      NPAI=NPA(I)
```

```
      DO 9050 J=1,NTAS
      IF(GN(I,J).GT.0.5)GO TO 9050
      JH=J
      GO TO 9112
 9050 CONTINUE
 9112 GHALF(I)=ETA(JH-1)+DETA(JH-1)*(GN(I,JH-1)-0.5)/(GN(I,JH-1)-
     1           GN(I,JH))
      CC(I)=CA*F6AG(I,1,2)/SQRT(X(NX)+XC)
 9000 CONTINUE
C**** TO OUTPUT INSTANTANEOUS CENTRE-LINE VEL.AND JET WIDTH
      WRITE(LP,8200)
      WRITE(LP,8300)(CC(I),I=1,NT1)
      WRITE (LP,8400)
      WRITE(LP,8300)(GHALF(J),J=1,NT1)
      WRITE(LP,8600)(NPT(J),J=1,NT1)
C**** TO OUTPUT MEAN VELOCITY PROFILES
      IF(NX.GT.NPRINT.OR.NX.EQ.1)GO TO 6111
      IF((NX+1).EQ.NXG(IPP).OR.(NX+2).EQ.NXG(IPP))GO TO 6111
      IF(JJ.NE.IFREQ)GO TO 6200
      JJ=0
 6111 CONTINUE
      WRITE (LP,6400)
      WRITE(LP,6700)
      DO 6200 J=1,JW,INC
 6200 WRITE(LP,6800)J,ETA(J),(AVP(J,K),K=1,3),GNCF(J)
      DO 9150 J=1,JW
 9150 GHA(J)=ETA(J)/AGHALF
 6300 CONTINUE
      NX=NX+1
      IF(NX.GT.NXT)RETURN
C**** SHIFT PROFILES
      DO 6500 I=1,NT
      JJ=NPAC(I)
      DO 6500 J=1,JJ
      F(I,J,1)=F(I,J,2)
      C(I,J,1)=C(I,J,2)
      G(I,J,1)=G(I,J,2)
      TNC(I,J,1)=TNC(I,J,2)
 6500 CONTINUE
      WRITE (LP,5250)
      RETURN
C     ------------------------------------
 5250 FORMAT(20X,27H***************************)
 6400 FORMAT(///23H MEAN VELOCITY PROFILE:/)
 6600 FORMAT(/5H NX= ,I4,2X,7H X(NX)=,F8.4,2X,4HINT=,I3,2X,4HJJ= ,I3,2X,
     1           5HUMAX=,E12.5,2X,6HCLUC=,E12.5)
 6700 FORMAT(///2X,1HJ,7X,3HETA,9X,1HF,15X,1HC,14X,1HG,12X,4HGNCF//)
 6800 FORMAT(I3,2X,F8.3,4(2X,E12.5))
 6900 FORMAT(//10X,20HAVE CENTRE-LINE UC= ,E15.7/10X, 27HAVE NO OF TRANS
     1 GRID PTS = ,I5/10X,27HMAX NO OF TRANS GRID PTS = ,I5/10X,27HMIN N
     2O OF TRANS GRID PTS = ,I5)
 6920 FORMAT(10X,20HAVE JET HALF WIDTH =,E14.7)
 6950 FORMAT(10X,27HPEAK TO PEAK OSCILLATION = ,F6.2,9H PER CENT)
 8000 FORMAT(10X,19HPHASE DIFFERENCE = ,F9.4,3H DEGREES)
 8100 FORMAT(//33H INSTANTANEOUS VELOCITY PROFILES:)
 8120 FORMAT(///2X,1HJ,3X,3HETA,9X,4(4HGNCF,10X,1HG,12X))
 8130 FORMAT(I3,2X,F8.5,4(1X,E12.5))
 8200 FORMAT(36H INSTANTANEOUS CENTRE-LINE VELOCITY:)
 8300 FORMAT(8(2X,E12.5))
 8400 FORMAT(30H INSTANTANEOUS JET HALF-WIDTH:)
 8500 FORMAT(12F8.5)
 8600 FORMAT(51H INSTANTANEOUS TOTAL NO. OF TRANSVERSE GRID POINTS:/
     15X,17(I5,2X))
 9155 FORMAT(2I5,E14.7)
 9900 FORMAT(2X)
      END

      SUBROUTINE SMOOTH
      COMMON/BLC/LC,LP
      COMMON/BL1/NP,NPA(51),NX,NXT,NT,DETA(100),A(100),ETA(100),IFREQ,
     1           INC
      COMMON/BL3/CLUC(100),F(51,100,2),C(51,100,2),G(51,100,2)
      COMMON/BL12/NPAC(51)
      COMMON/BL13/NT1
```

```
      COMMON/ALSCC/NEW
      LOGICAL LOGMG
C     -----------------------------------
C**** THIS SMOOTHES THE VELOCITY GRADIENT PROFILE
C
      IF(NX.LT.NTH)GO TO 3800
      DO 3500 I=1,NT
      ISM1=1
      ISM2=1
      JJ=NSTC(I)
      DO 3130 J=1,JJ
      LOGMG=Q(I,J,2).LT.0..AND.Q(I,J+2,2).LT.0.
      IF(Q(I,J,2).GT.Q(I,J+1,2).AND.Q(I,J+2,2).GT.Q(I,J+1,2).AND.
     1Q(I,J+1,2).LT.-1..AND.LOGMG)GO TO 3120
      GO TO 3130
 3120 JI=J+1
      GO TO 3150
 3130 CONTINUE
 3150 CONTINUE
      JII=JI+1
      DO 3170 J=JII,JJ
      IF(Q(I,J,2).GT.Q(I,J-1,2).AND.Q(I,J,2).LT.0.)GO TO 3170
      ISM1=J-1
      JI=J
      QCOMP=Q(I,ISM1,2)
      GO TO 3180
 3170 CONTINUE
      IF(ISM1.EQ.1)GO TO 3460
 3180 DO 3300 J=JI,JJ
      IF(Q(I,J,2).LE.QCOMP.OR.Q(I,J,2).GT.0.)GO TO 3300
      ISM2=J
      GO TO 3335
 3300 CONTINUE
      IF(ISM2.EQ.1)GO TO 3460
 3335 CONTINUE
      GG=ETA(ISM2)-ETA(ISM1)
      GR=Q(I,ISM1,2)-Q(I,ISM2,2)
      FF=GR/GG
      JI=ISM1+1
      JF=ISM2-1
      DO 3450 J=JI,JF
      FACT=ETA(J)-ETA(ISM1)
 3450 Q(I,J,2)=Q(I,ISM1,2)-FACT*FF
      WRITE(LP,3470)I,ISM1,ISM2
 3460 CONTINUE
 3500 CONTINUE
      WRITE(LP,4000)
 3800 RETURN
C     -----------------------------------
 3470 FORMAT(5X,4H I =,I5,2X,7H ISM1 =,I5,2X,7H ISM2 =,I5)
 4000 FORMAT(/5X,35H***** RETURNED FROM SMOOTHING *****/)
      END
```

Appendix B. Figures

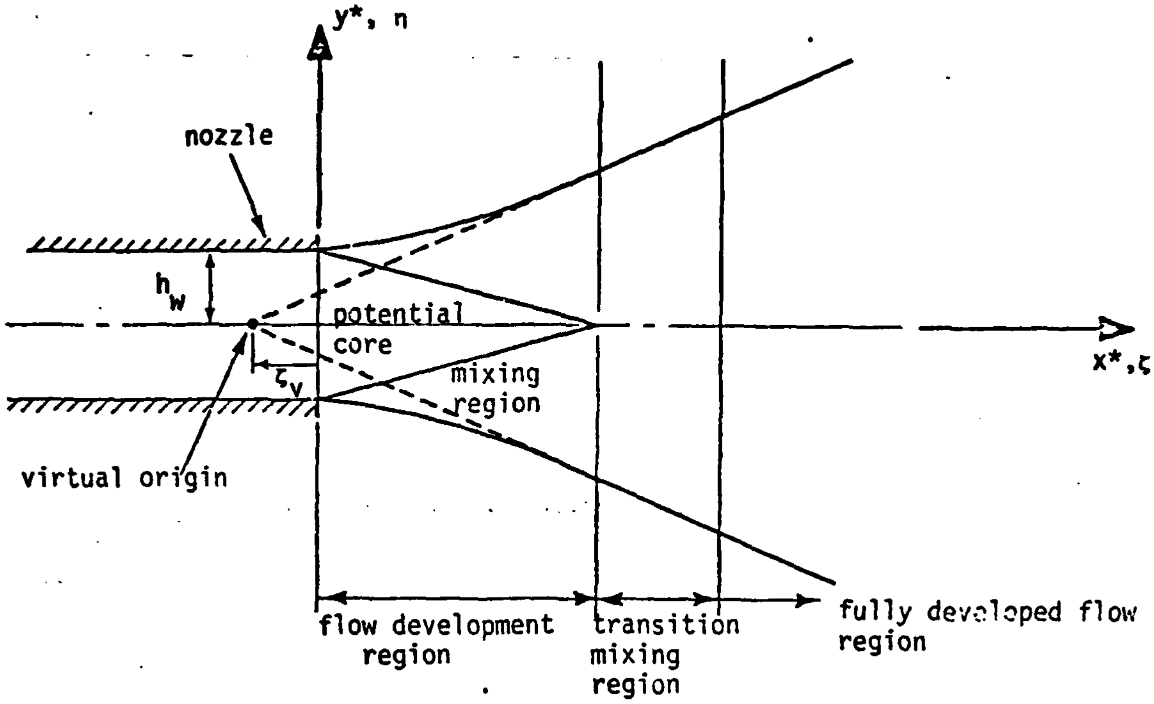

Figure 1. Cinfiguration of the Plane Jet

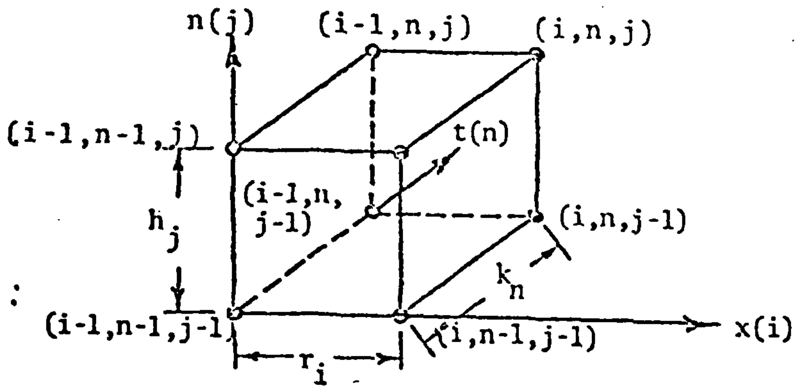

FIGURE 2. Net cube for the difference equations for Eq (21).

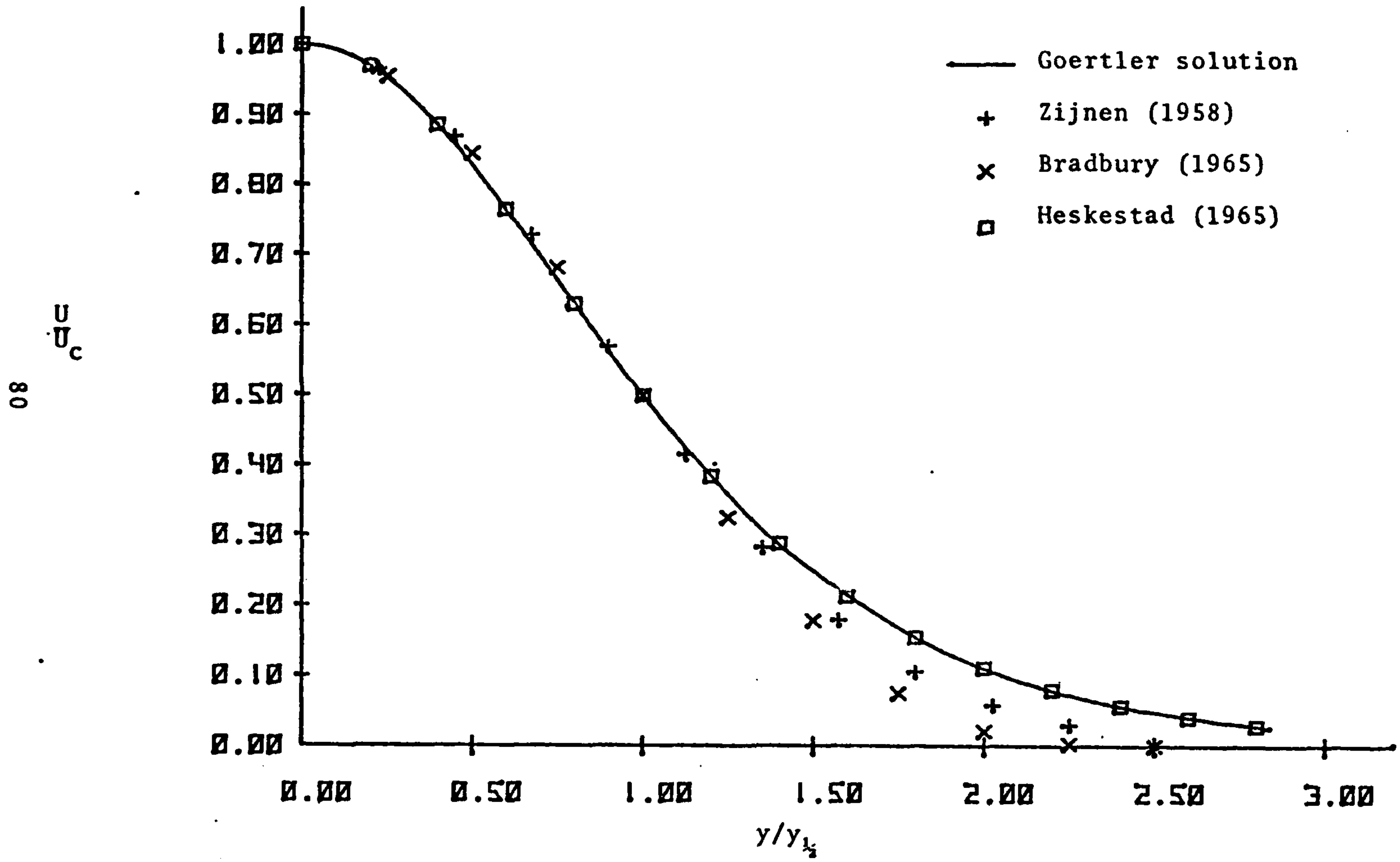

Figure 3. Non-Dimensional Self-Preserved Velocity Profile of the Steady Jet

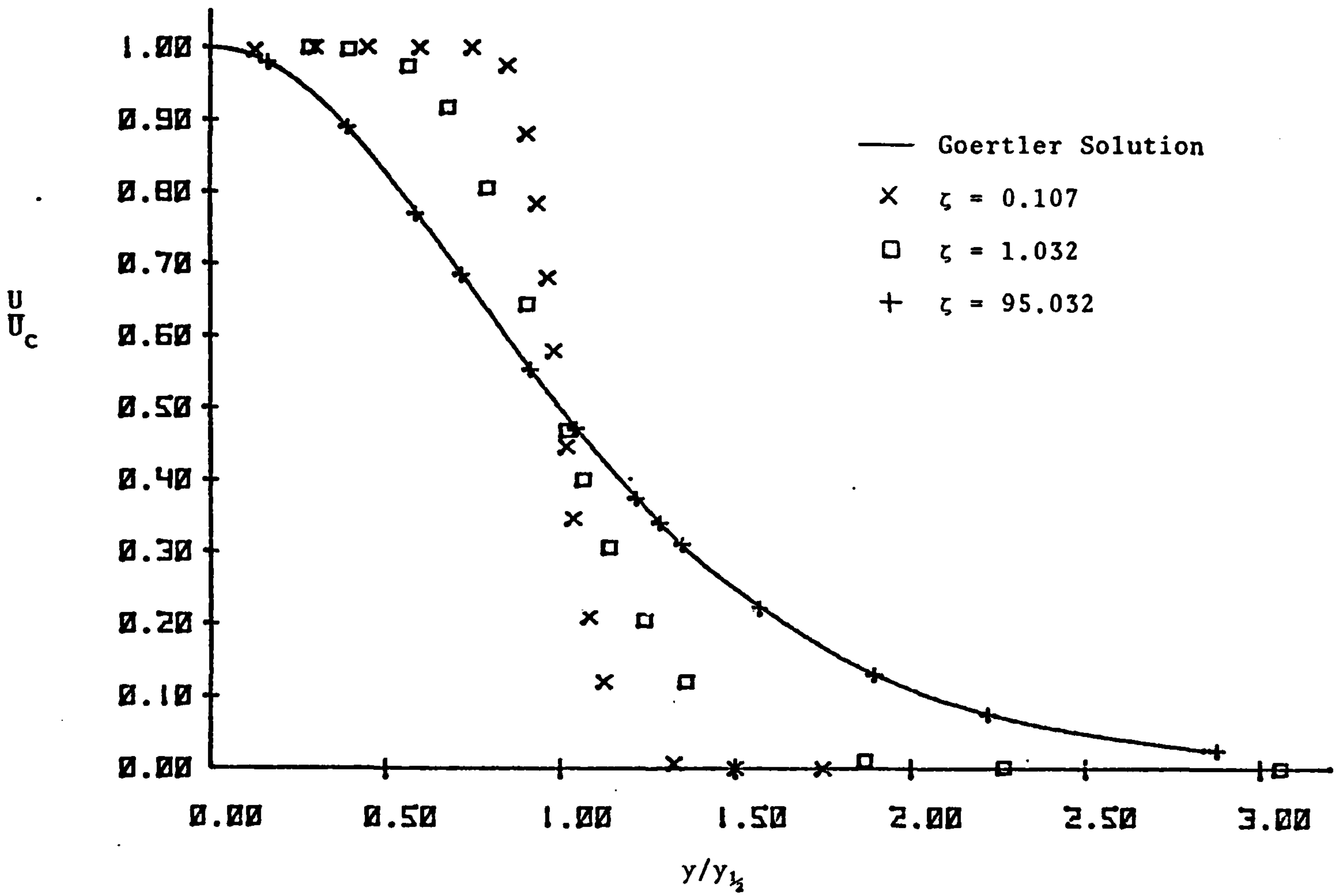

Figure 4(a) Non-Dimensional Velocity Distribution of the Steady Jet

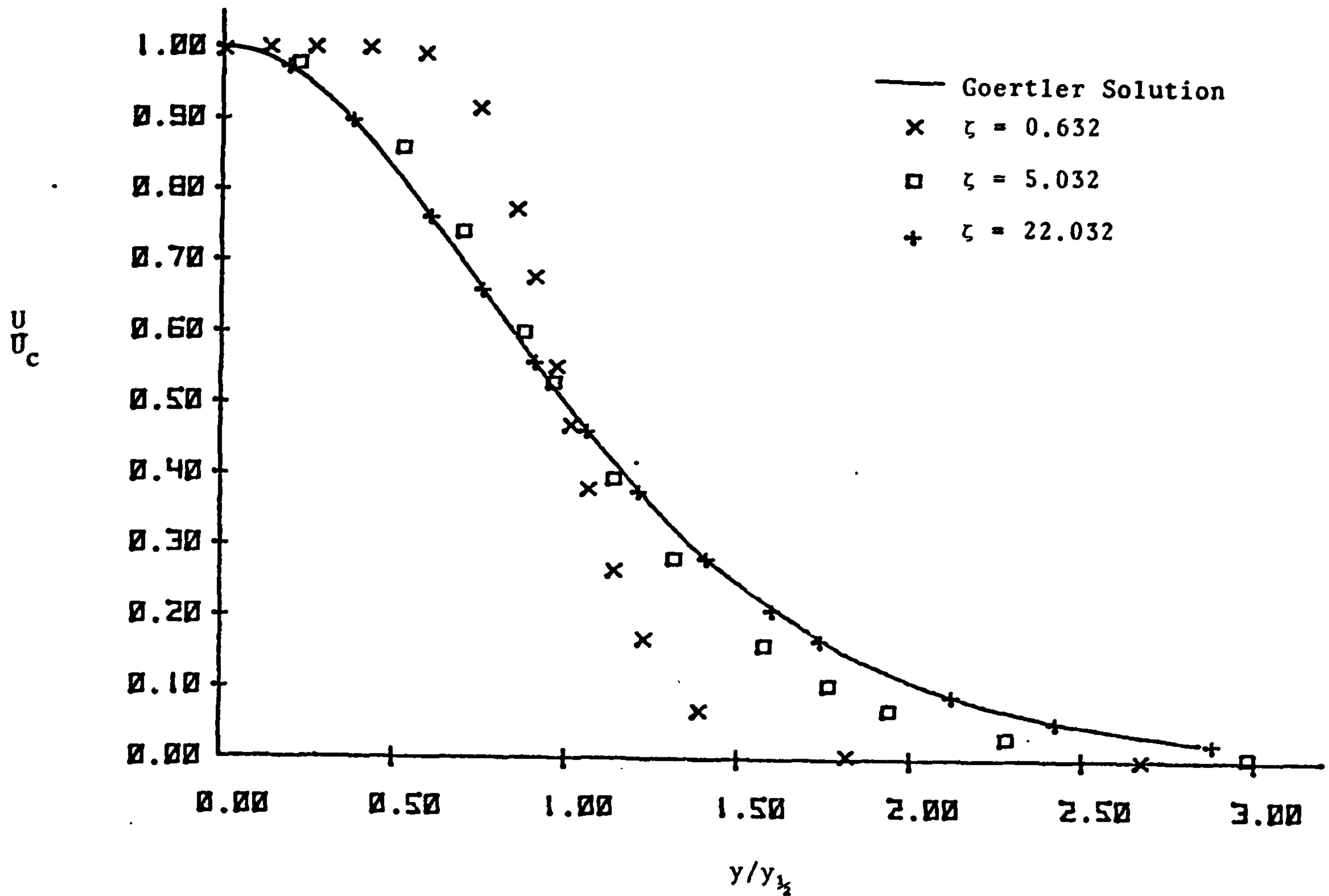

Figure 4(b) Non-Dimensional Velocity Distribution of the Steady Jet

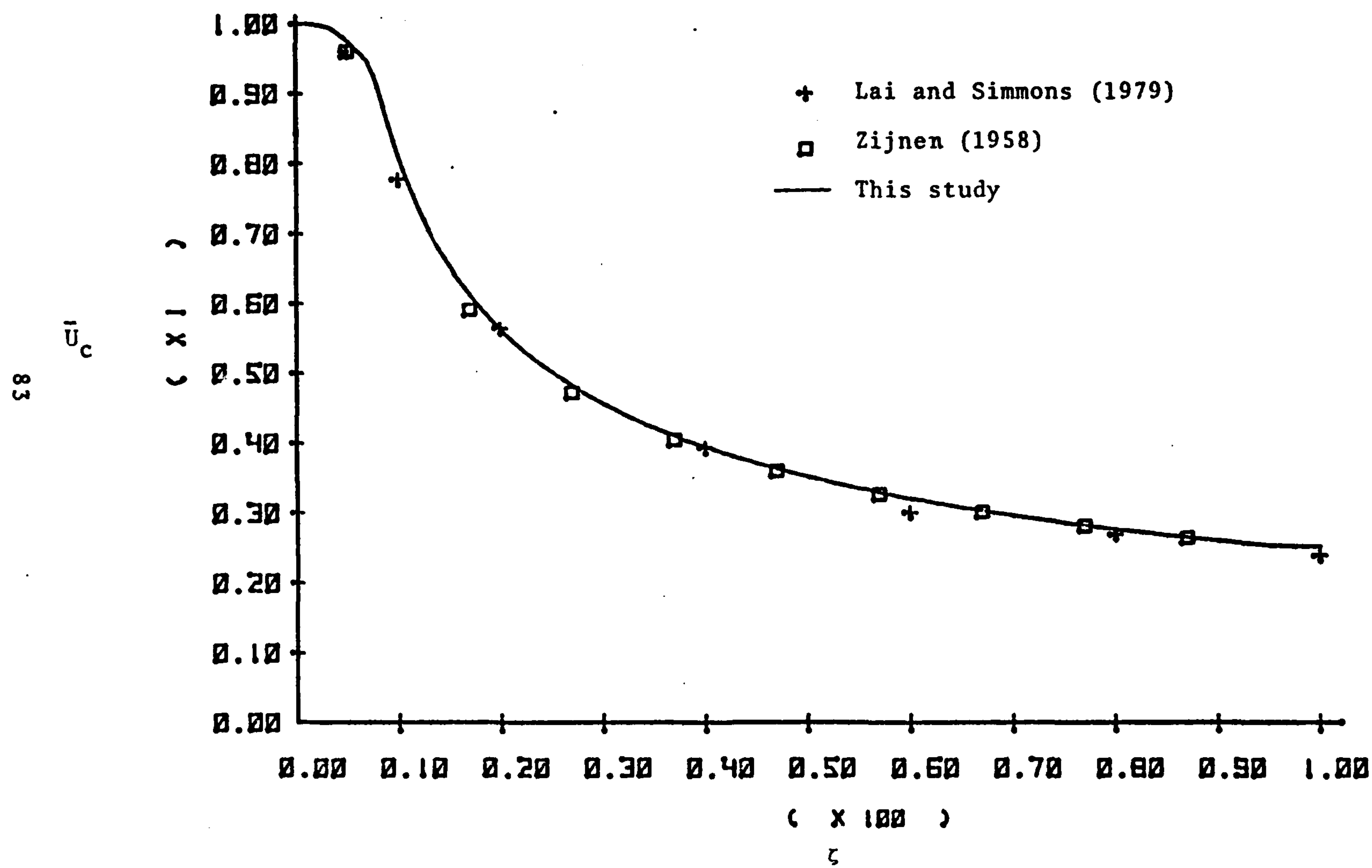

Figure 5 Variation of Centre-Line Velocity of the Steady Jet with Streamwise Distance

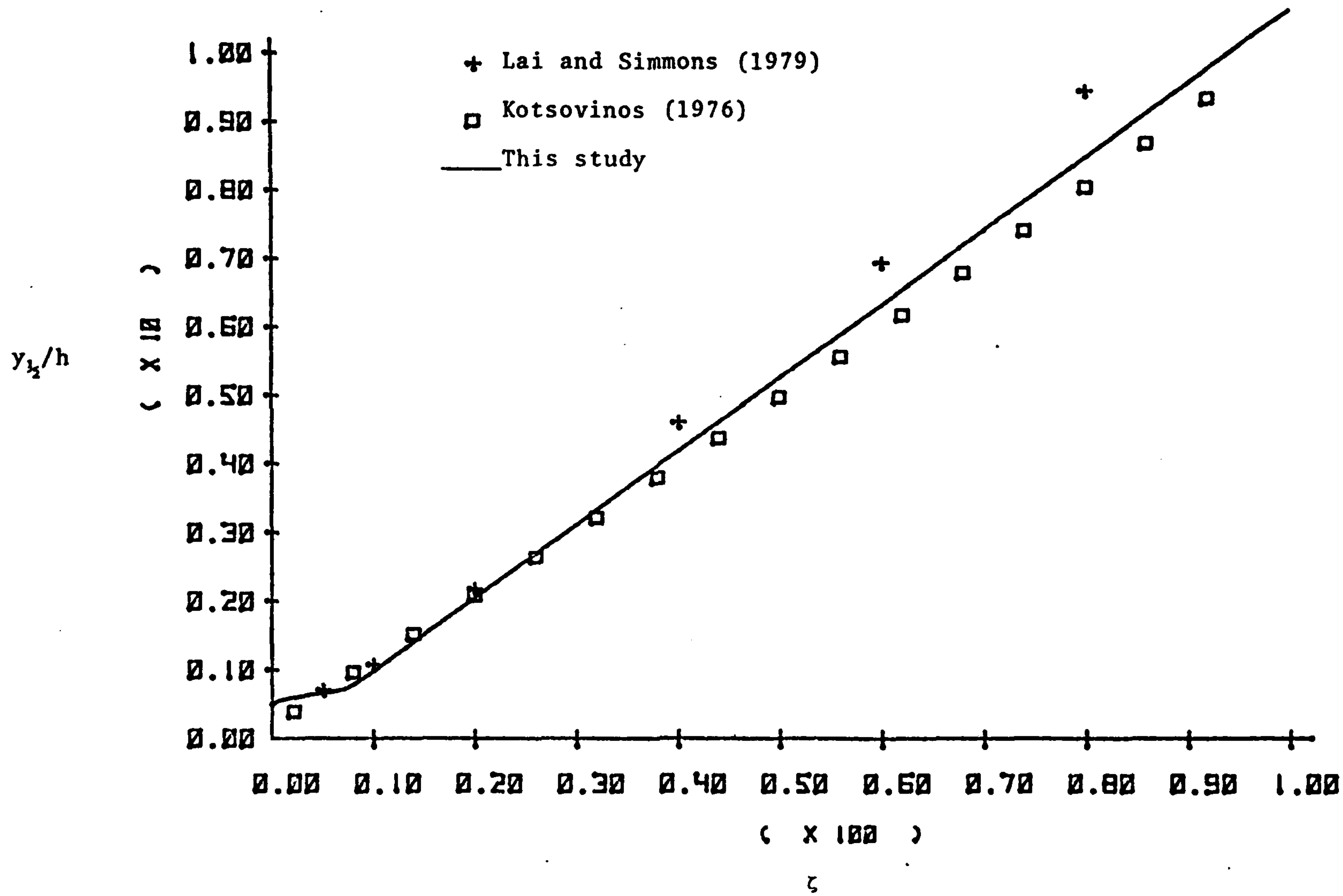

Figure 6 Variation of the Half-Width of the Steady Jet with Streamwise Distance

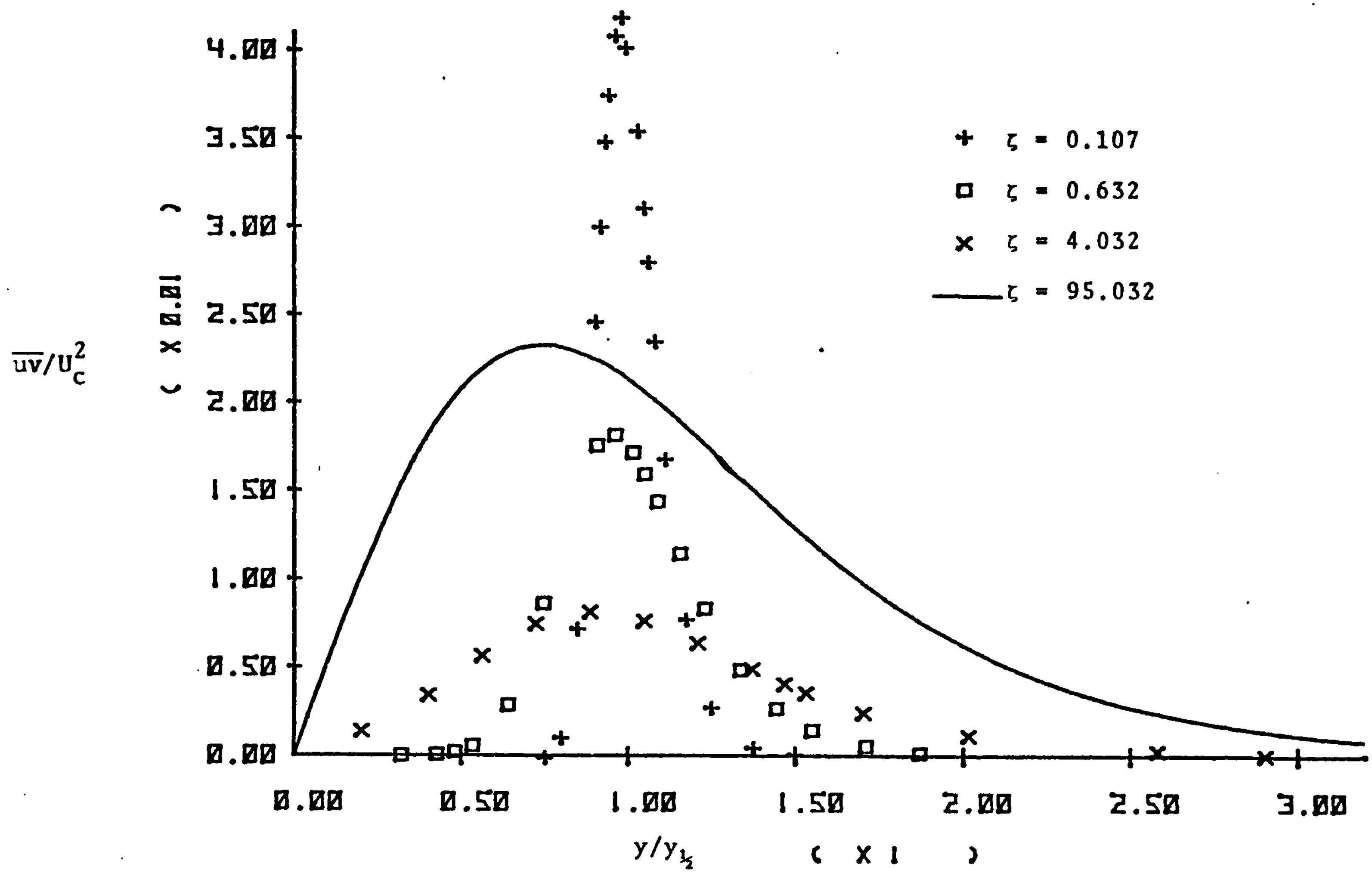

Figure 7(a) Nondimensional Shear Stress Profile for the Steady Jet

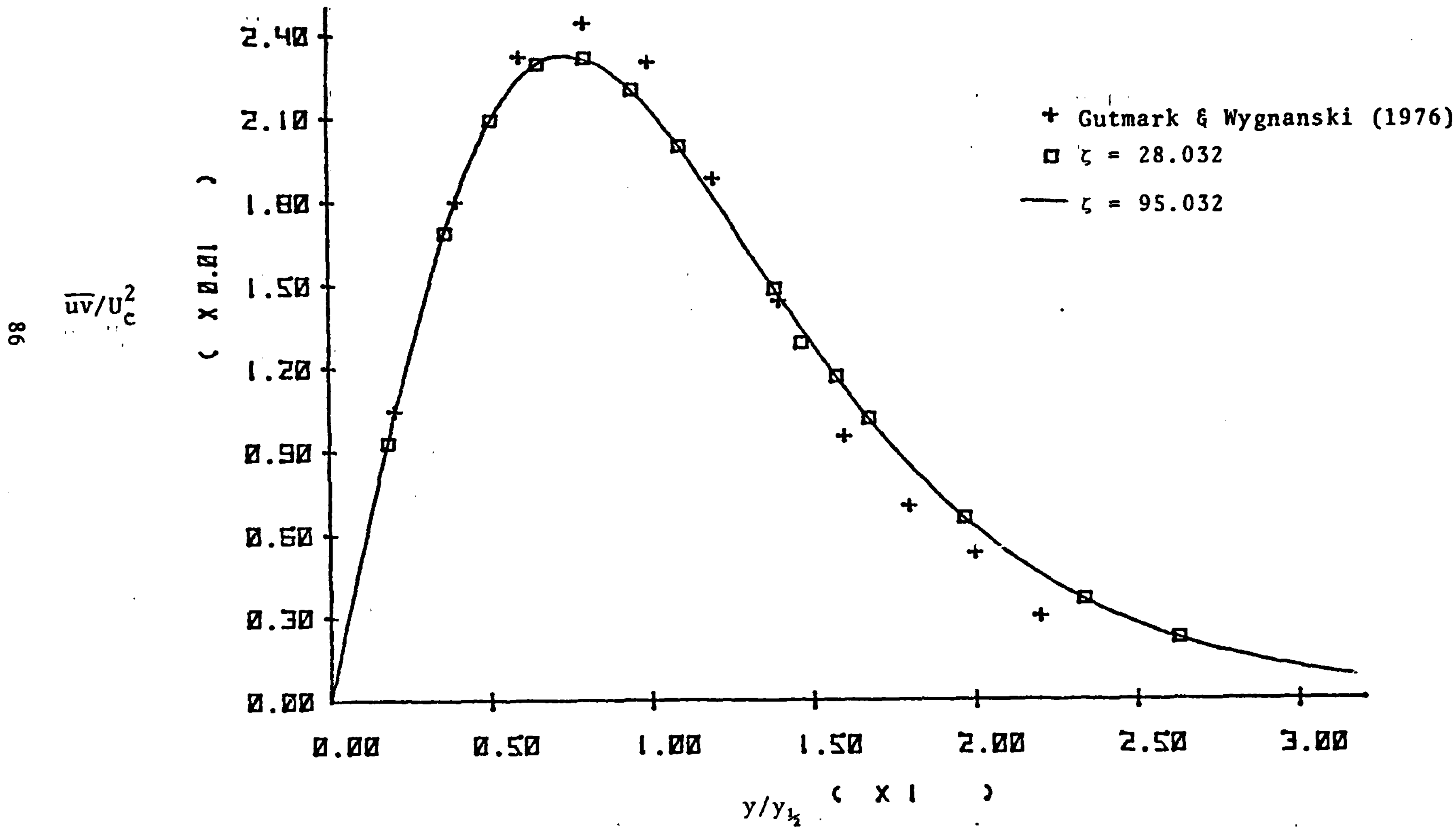

Figure 7(b) Nondimensional Shear Stress Profile for the Steady Jet

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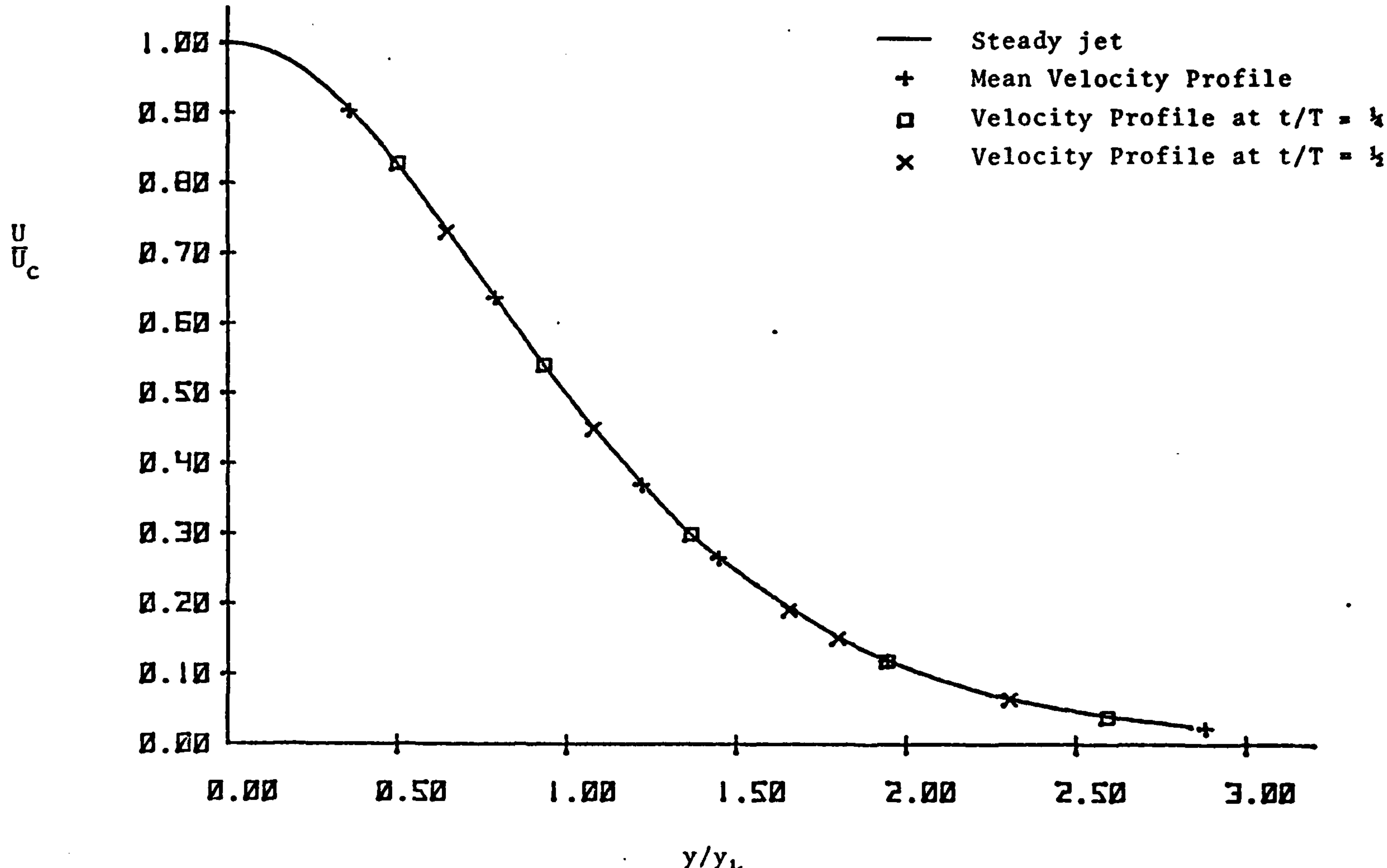

Figure 8(a) Non-dimensional Velocity Distribution for $\zeta = 40.032$, $\omega = 0.000871$, $\epsilon = 0.1$

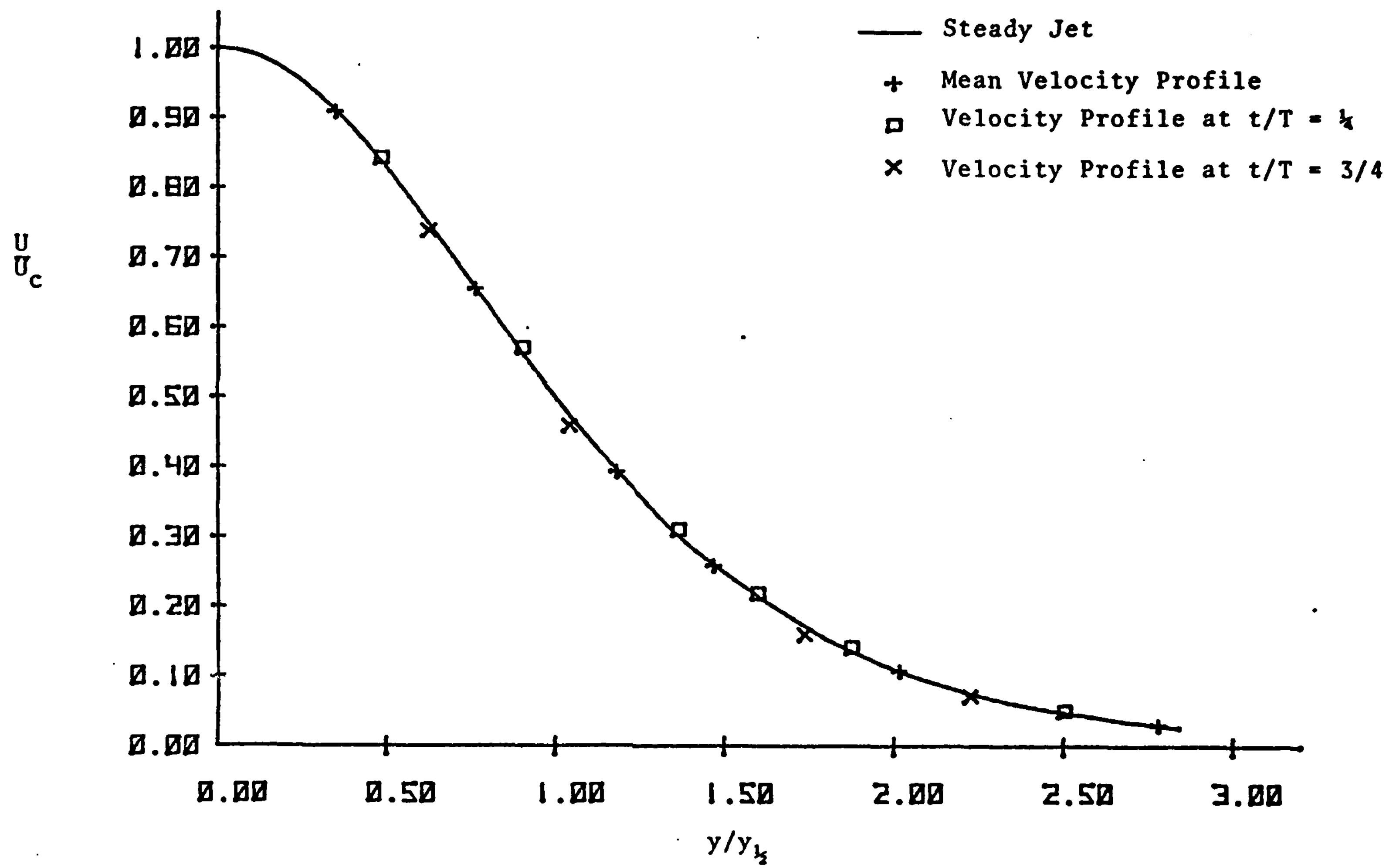

Figure 8(b) Non-dimensional Velocity Distritubion for ζ = 40.032, ω = 0.00871, ε = 0.1

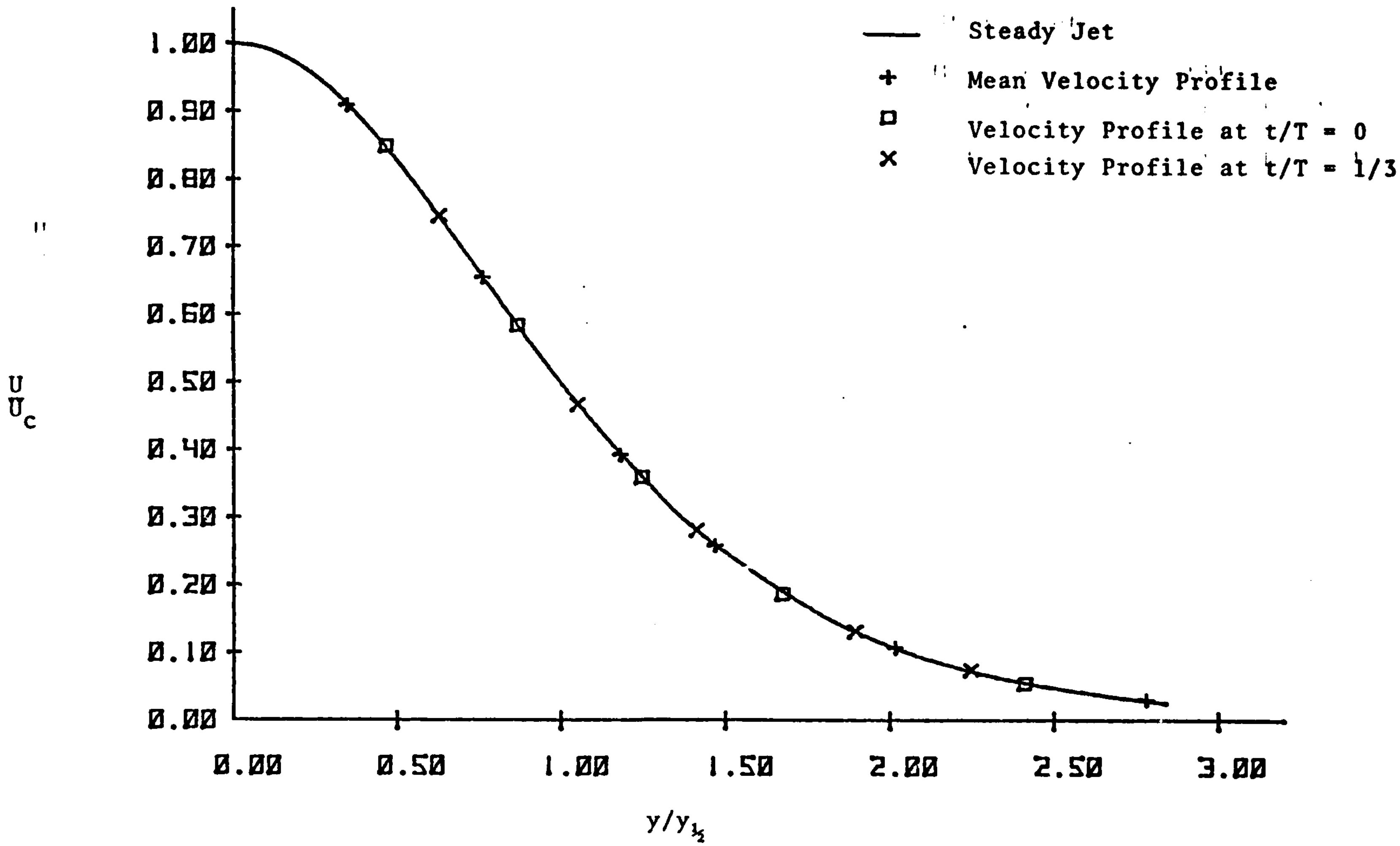

Figure 8(c) Non-dimensional Velocity Distribution for $\zeta = 40.032$

$\omega = 0.0871$

$\varepsilon = 0.1$

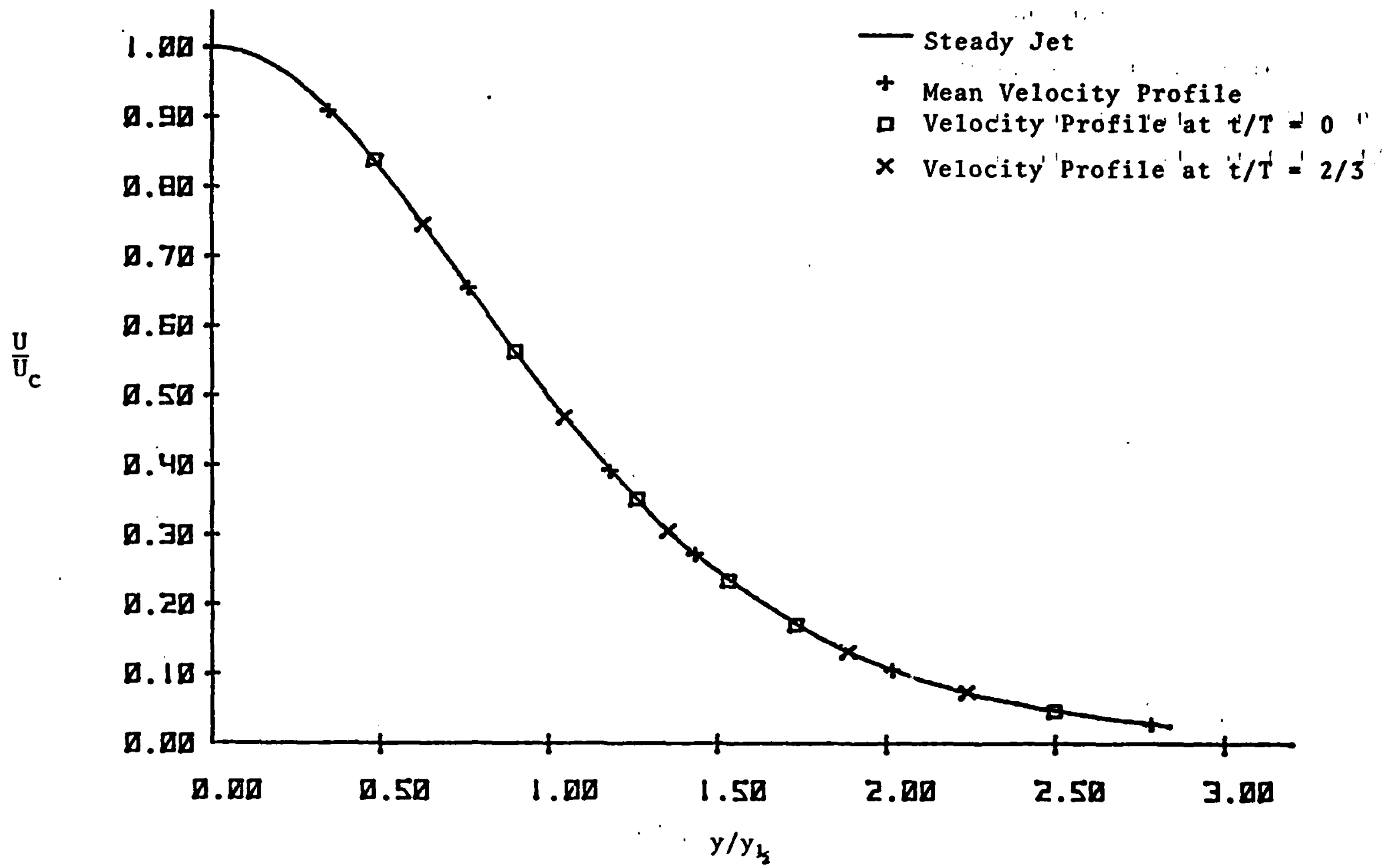

Figure 8(d) Non-dimensional Velocity distribution for $\zeta = 40.032$
$\omega = 0.000871$
$\epsilon = 0.15$

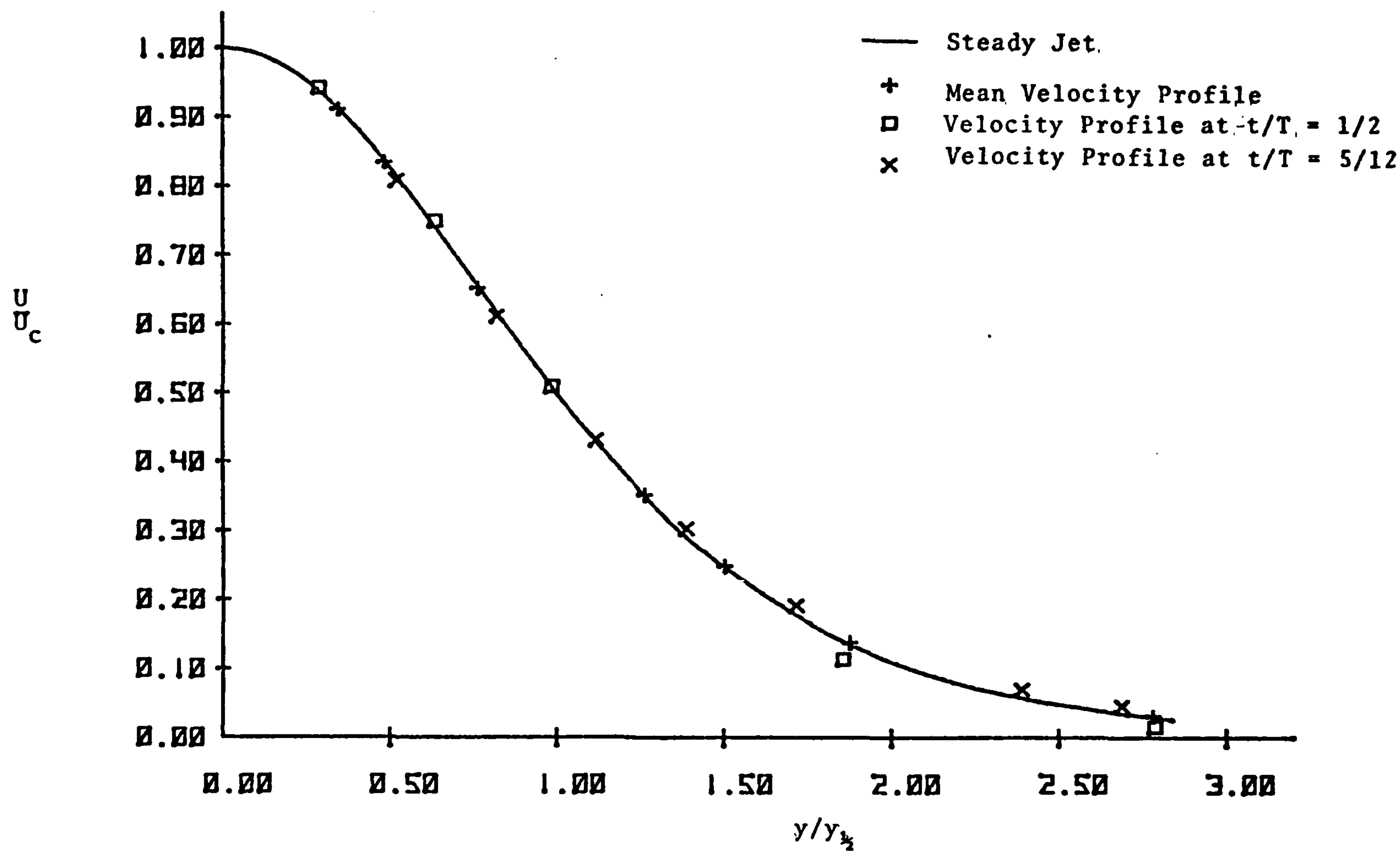

Figure 8(e) Non-dinensional Velocity Distribution for $\zeta = 40.032$
$\omega = 0.00871$
$\epsilon = 0.15$

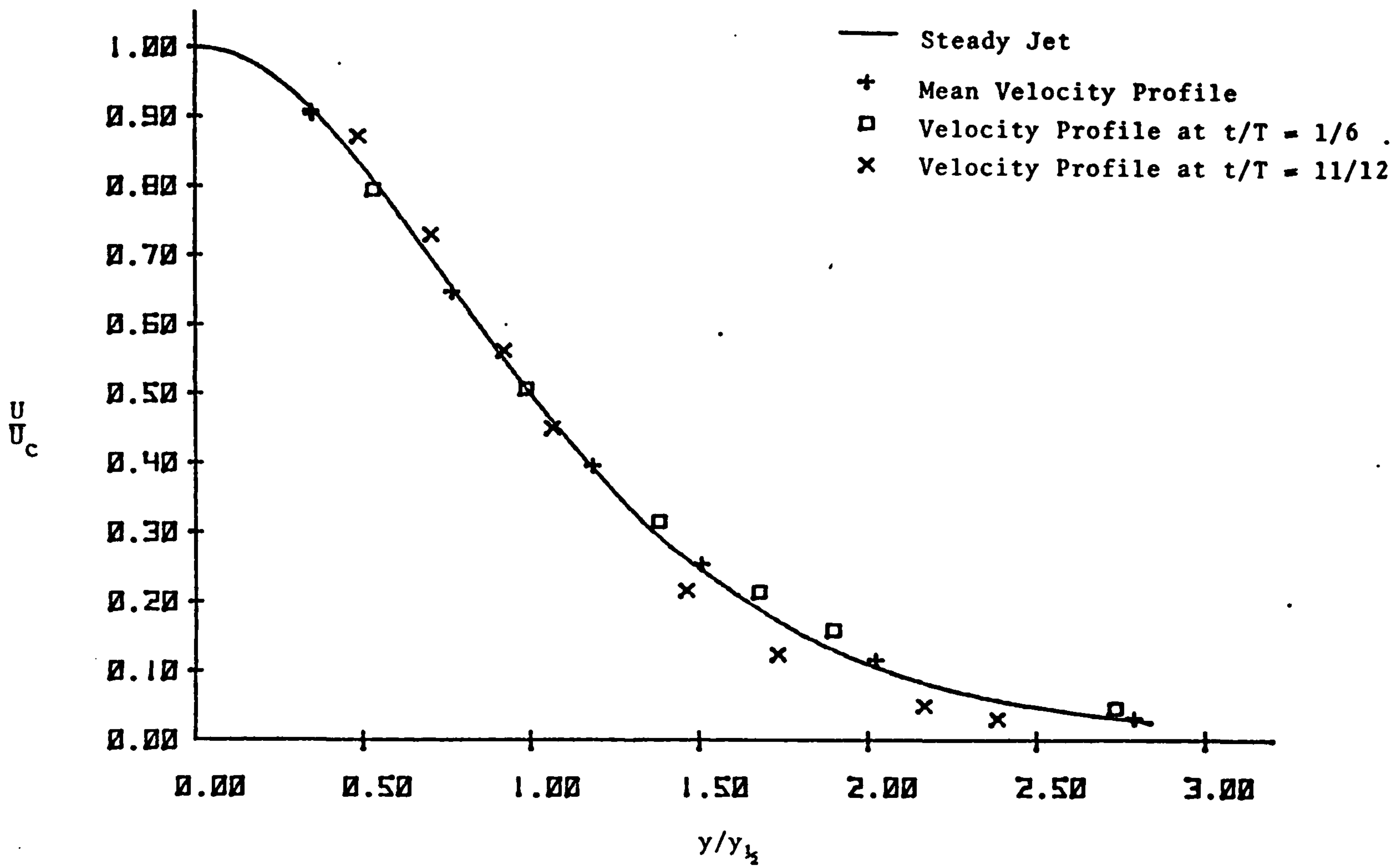

Figure 8 (f) Non-dimensional Velocity Distribution for $\zeta = 40.032$
$\omega = 0.0871$
$\epsilon = 0.15$

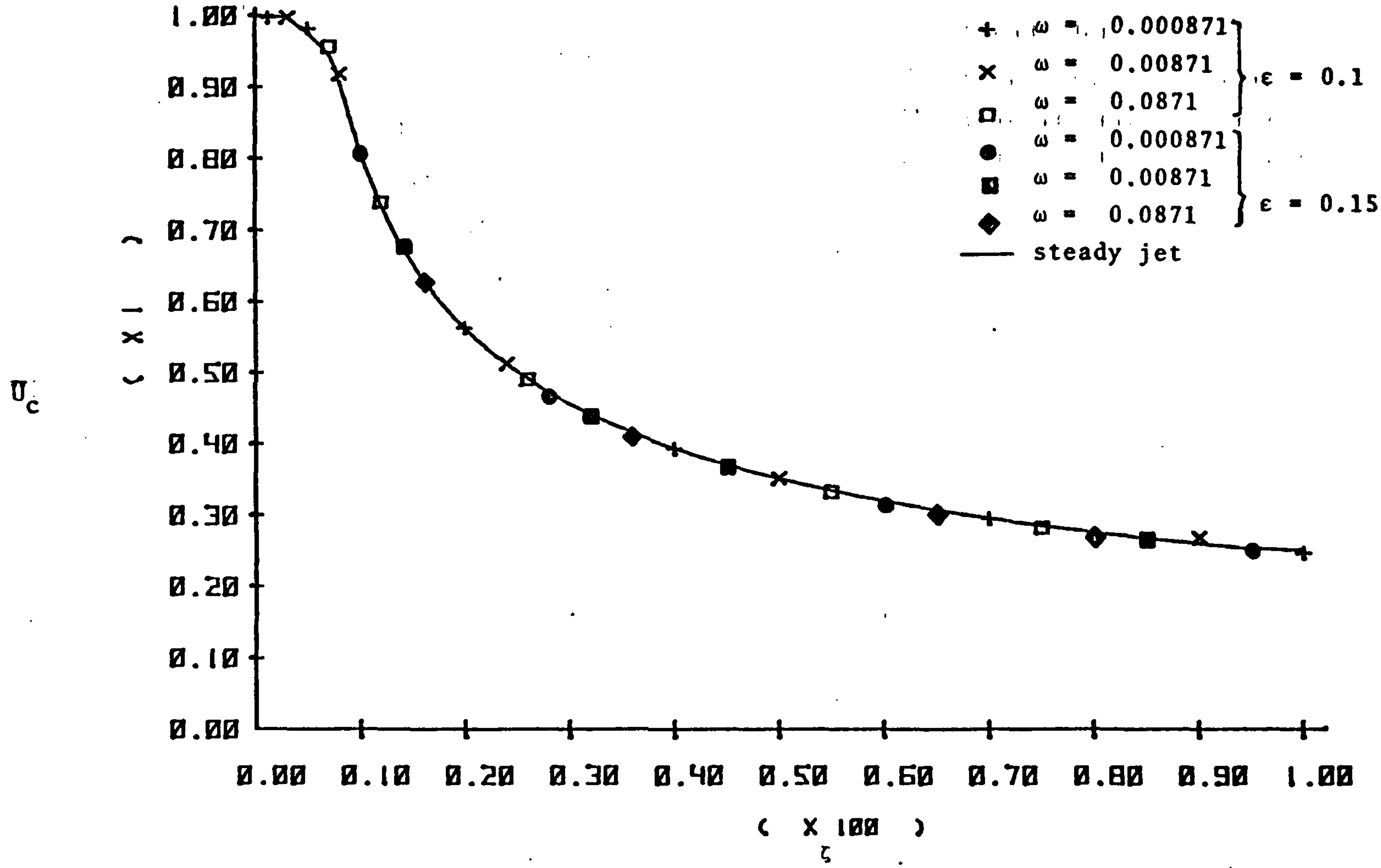

Figure 9 Variation of Mean Centre-Line Velocity with Streamwise Distance

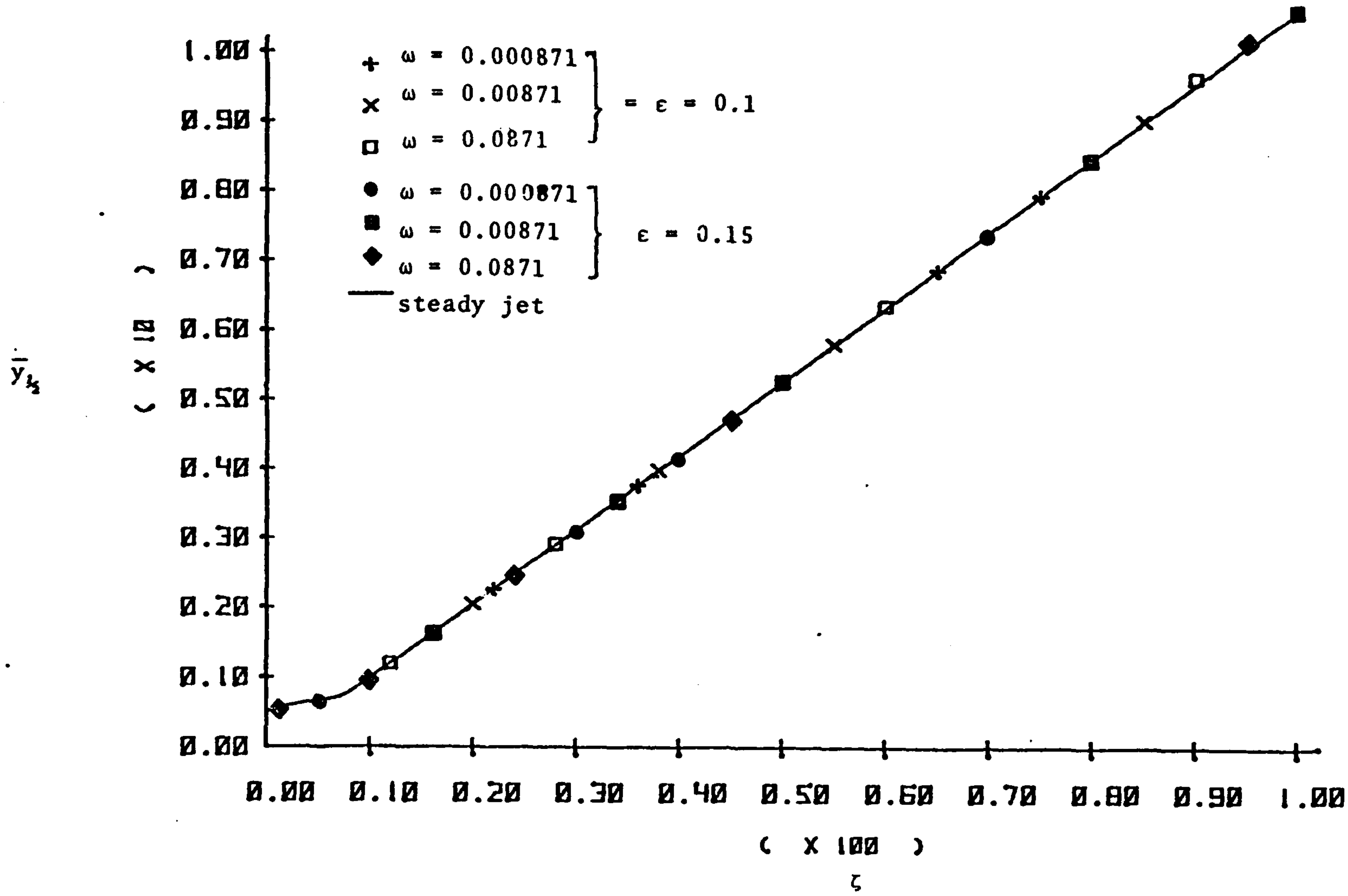

Figure 10 Variation of Mean Jet Half-with Streamwise Distance

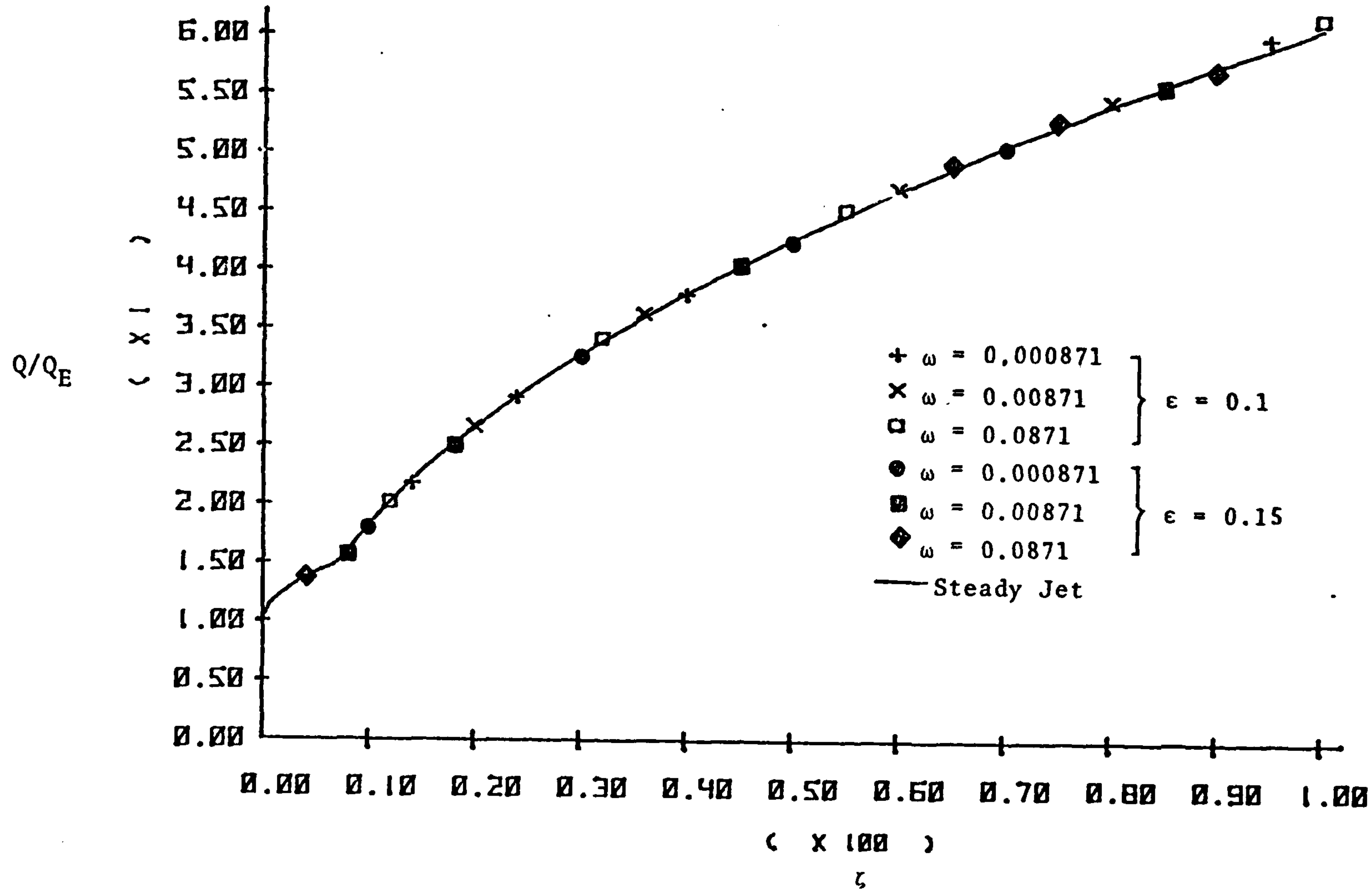

Figure 11 Variation of Non-Dimensional Mass Flow with Streamwise Distance

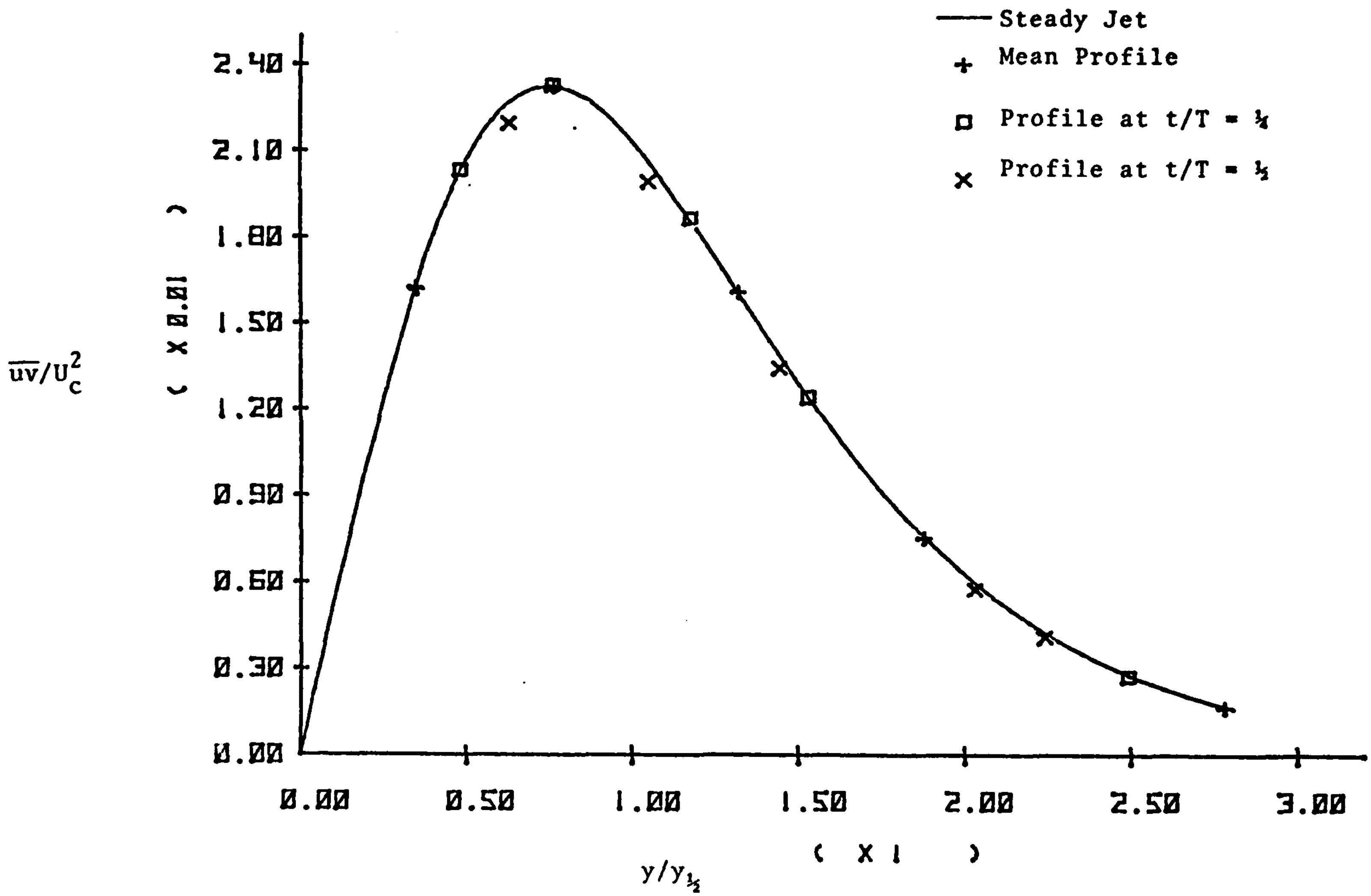

Figure 12(a) Non-Dimensional Shear Stress Profile for $\zeta = 40.032$, $\omega = 0.000871$, $\varepsilon = 0.1$

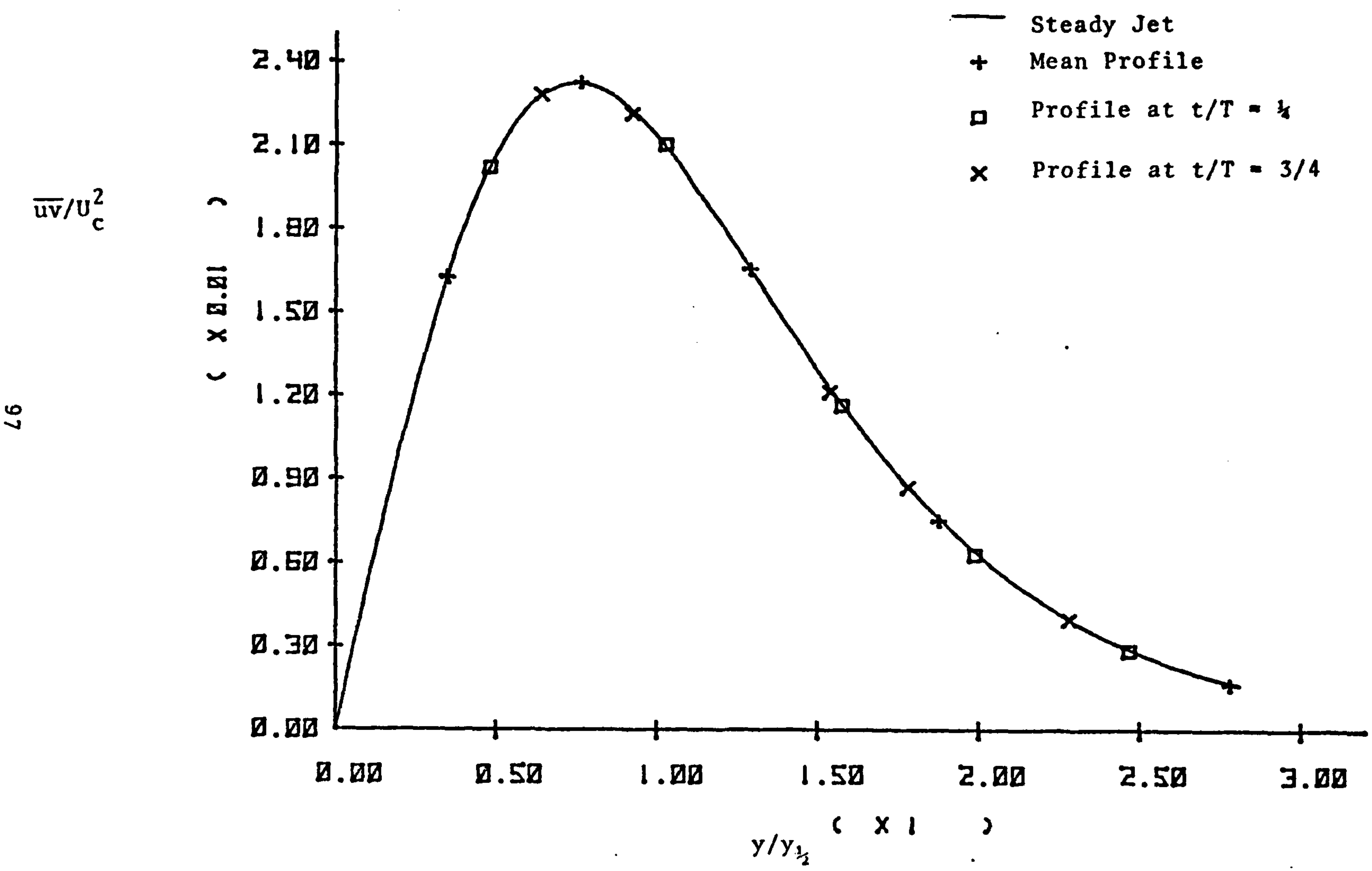

Figure 12(b) Non-dimensional Shear Stress Profile for ζ = 40.032, ω = 0.00871, ε = 0.1

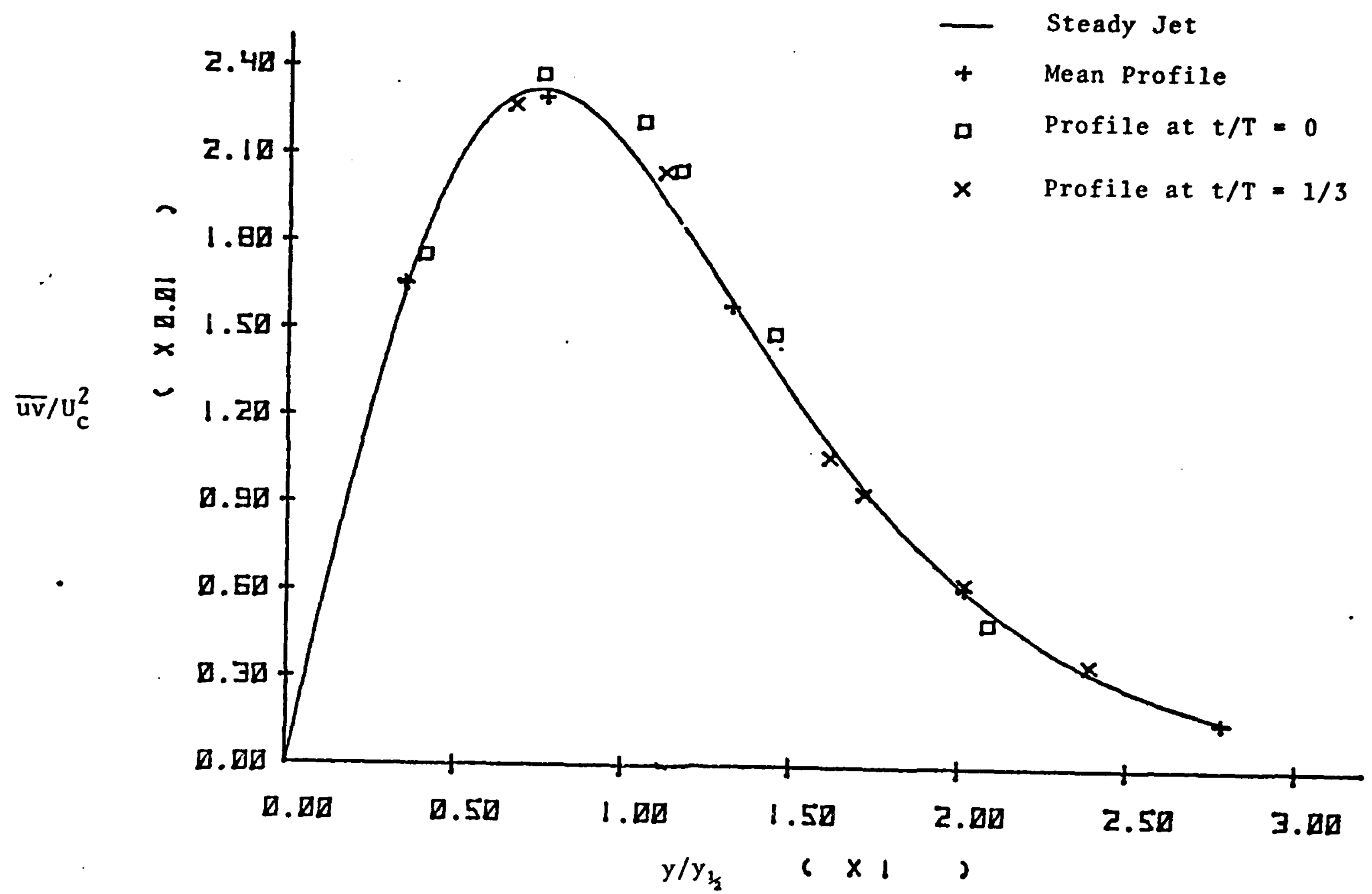

Figure 12(c) Non-dimensional Shear Stress Profile for ζ = 40.032, ω = 0.0871, ε = 0.1

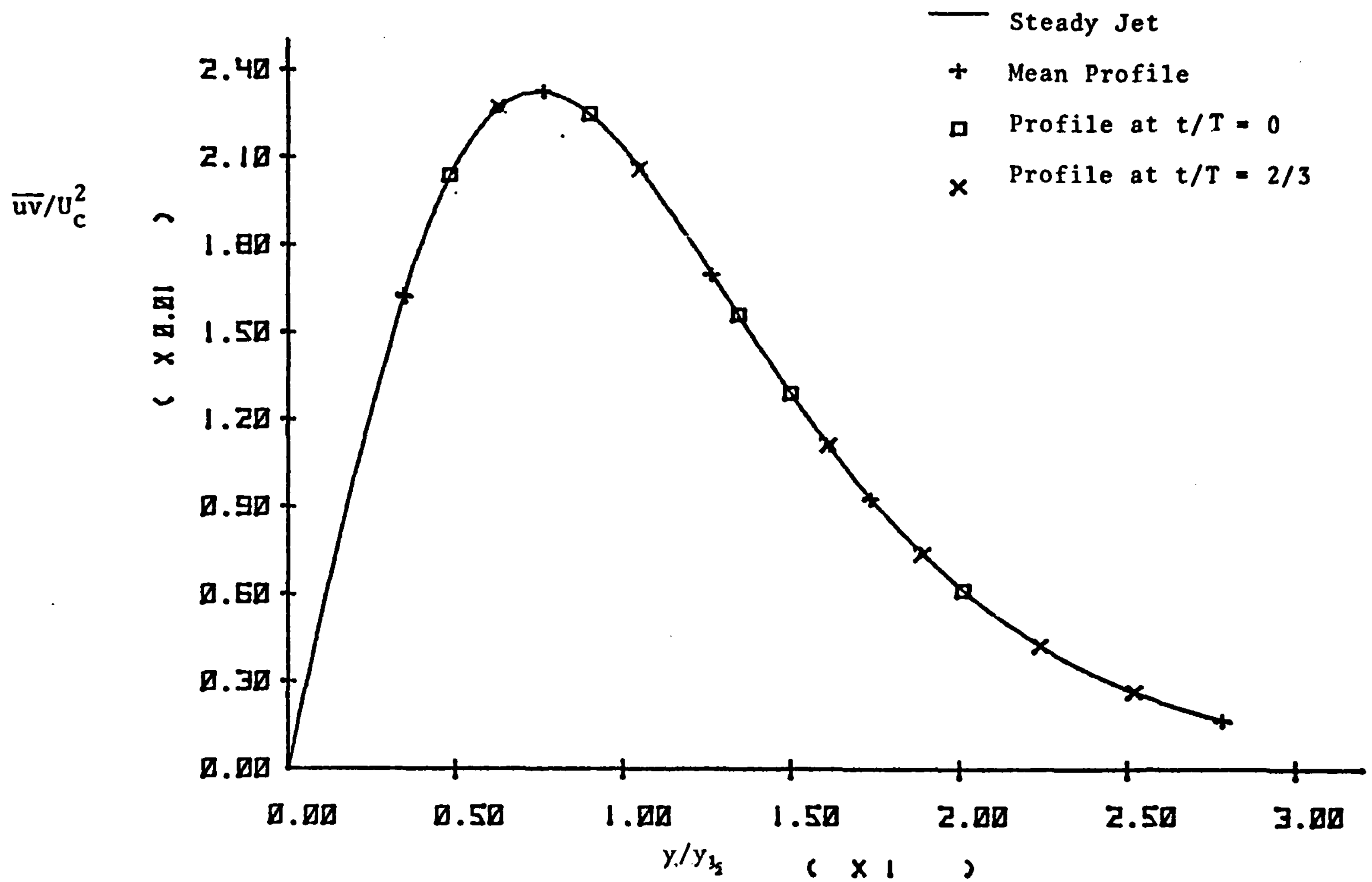

Figure 12(d) Non-Dimensional Shear Stress Profile for ζ = 40.032, ω = 0.000871, ε = 0.15

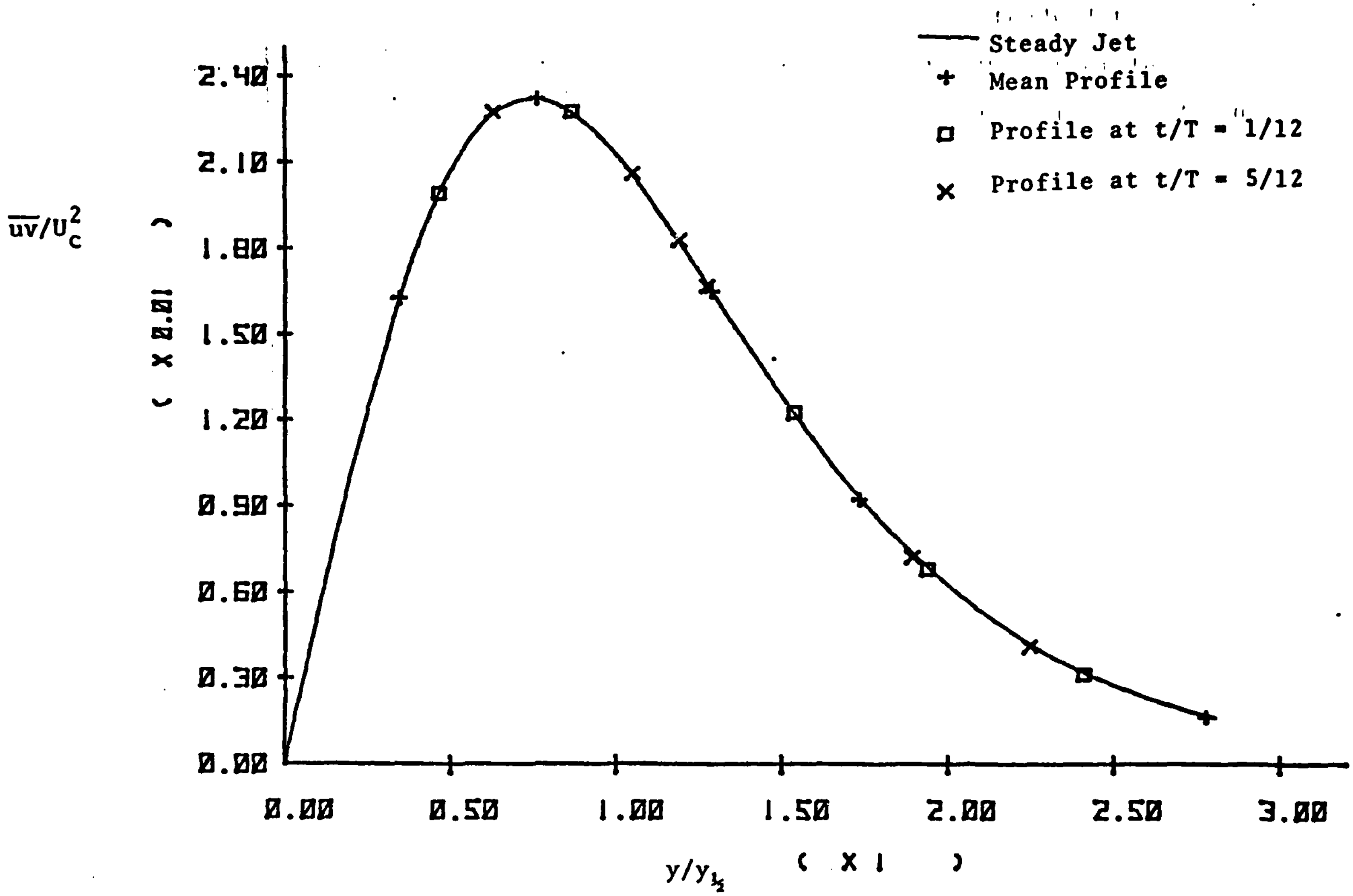

Figure 12(e) Non-Dimensional Shear Stress Profile for ζ = 40.032, ω = 0.00671, ϵ = 0.15

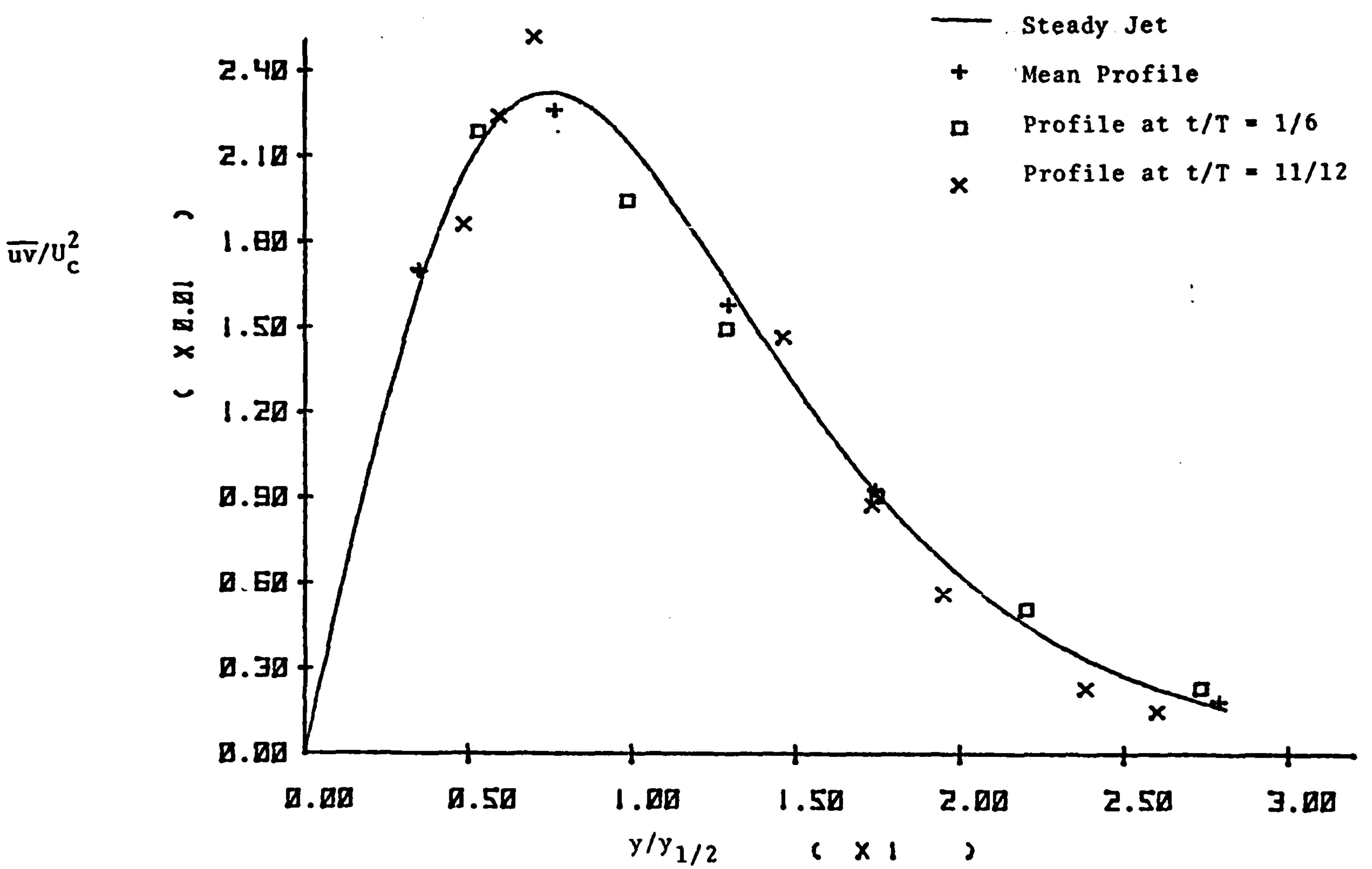

Figure 12(f) Non-Dimensional Shear Stress Profile for $\zeta = {}^{4}0.032$, $\omega = 0.0871$, $\varepsilon = 0.15$

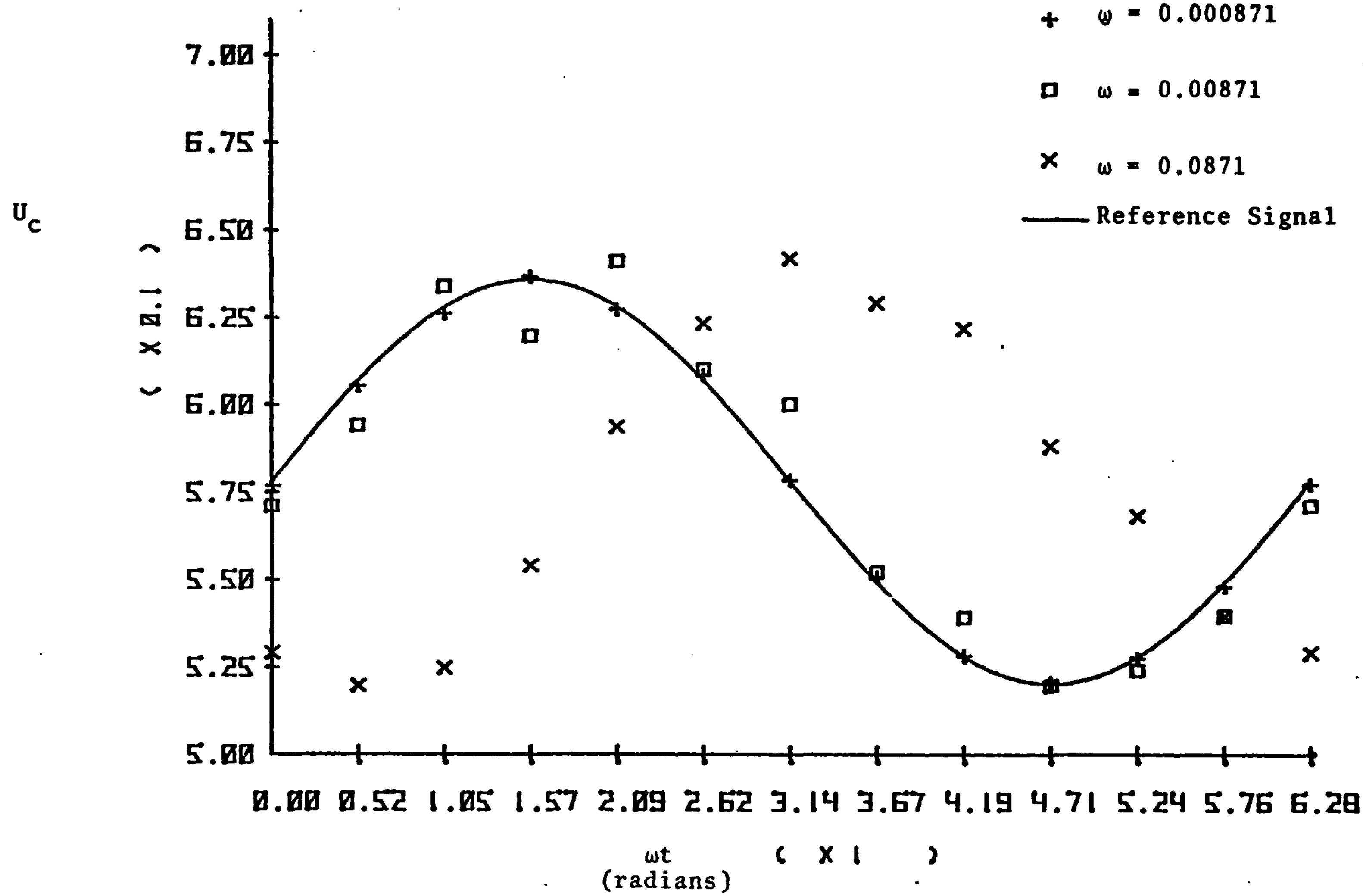

Figure 13(a) Variation of Instantaneous Centre-Line Velocity with Time for $\zeta = 19.032$, $c = 0.1$

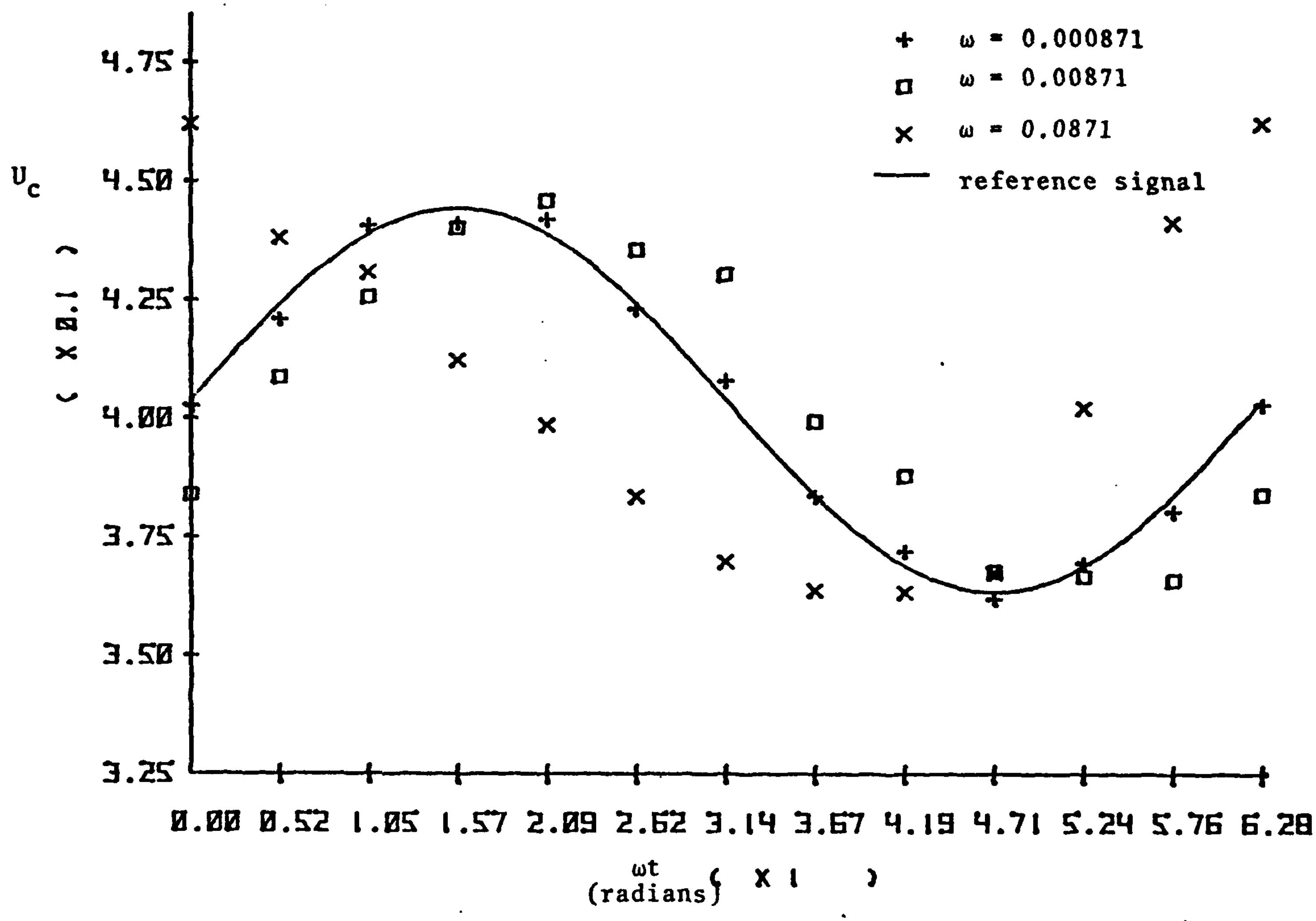

Figure 13(b) Variation of Instantaneous Centre-Line Velocity with Time for $\zeta = 38.032$, $\varepsilon = 0.1$

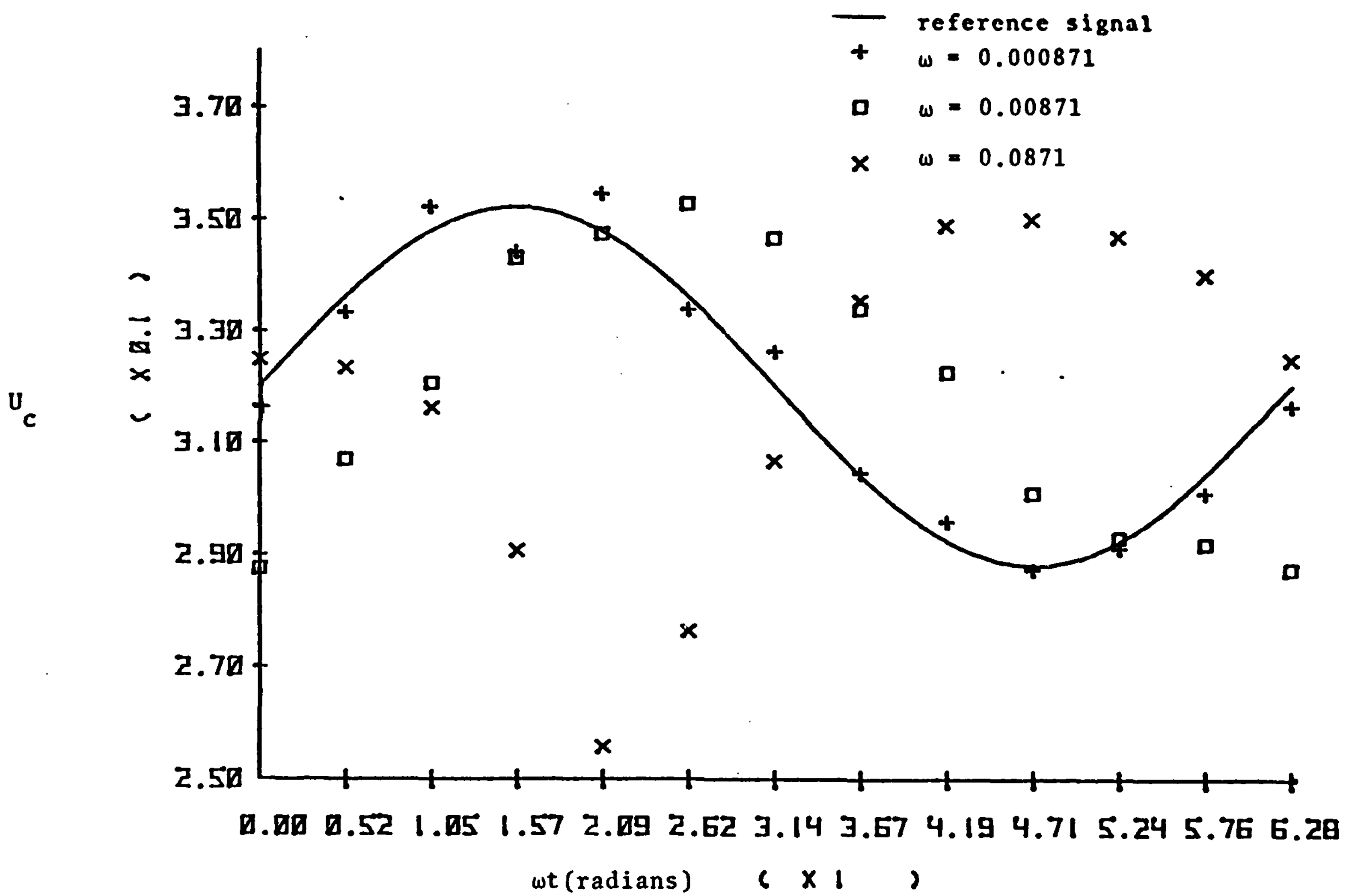

Figure 13(c) Variation of Instanteneous Centre-Line Velocity with Time for $\zeta = 60.032$, $\epsilon = 0.1$

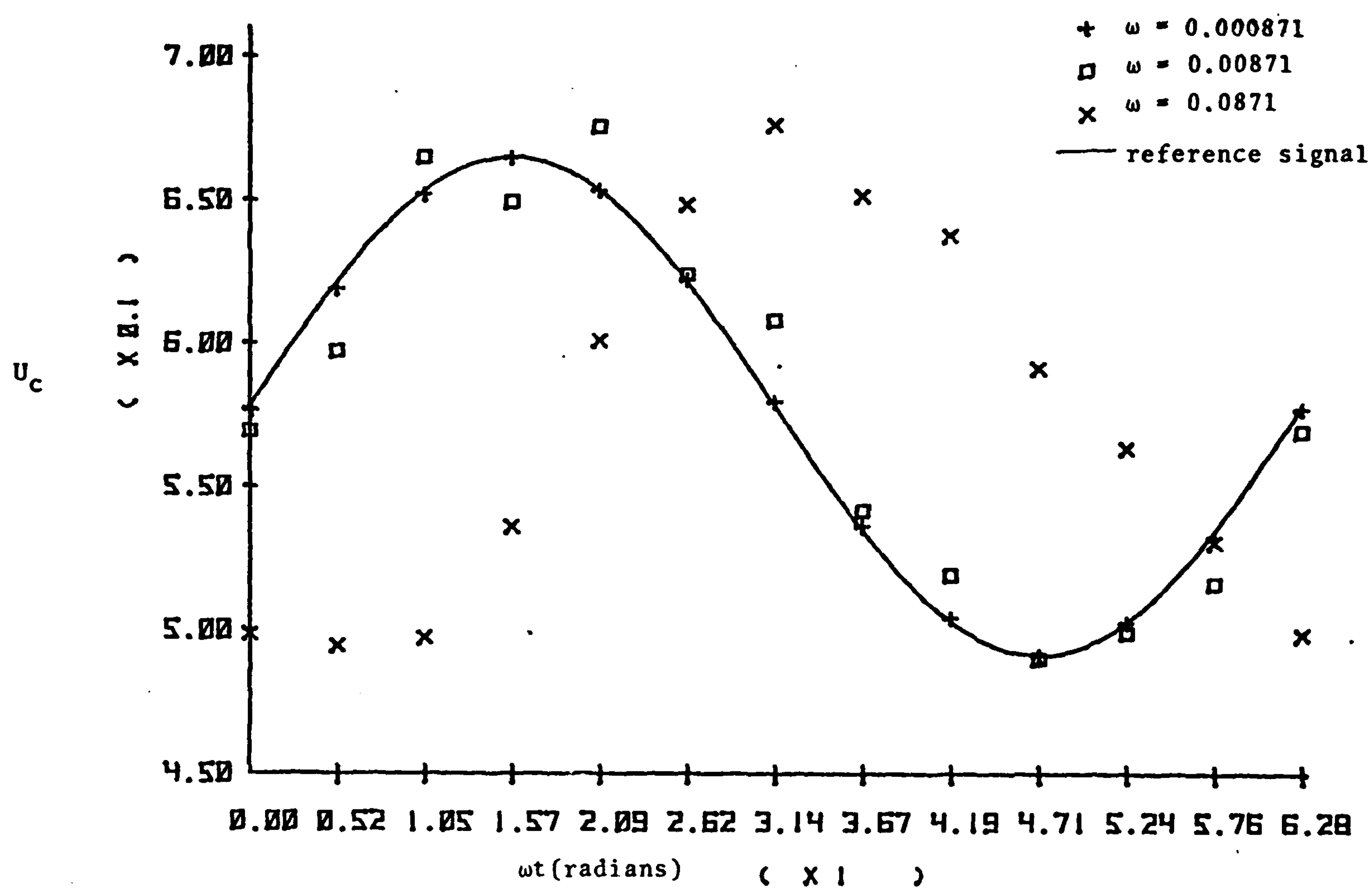

Figure 13(d) Variation of Instantaneous Centre-Line Velocity with Time for $\zeta = 19.032$, $\epsilon = 0.15$

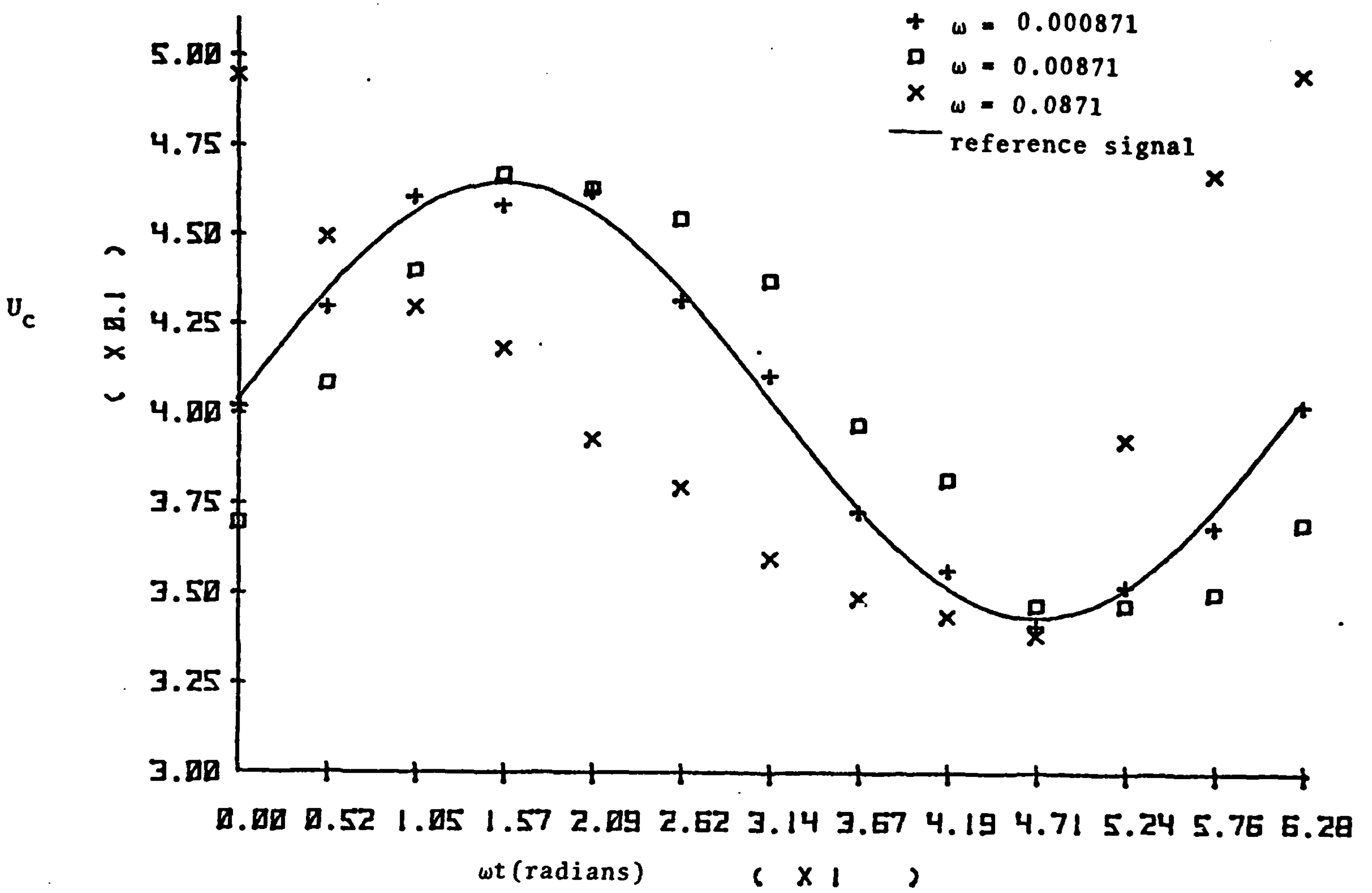

Figure 13(e) Variation of Instantaneous Centre-Line Velocity with Time for $\zeta = 38.032$, $\varepsilon = 0.15$

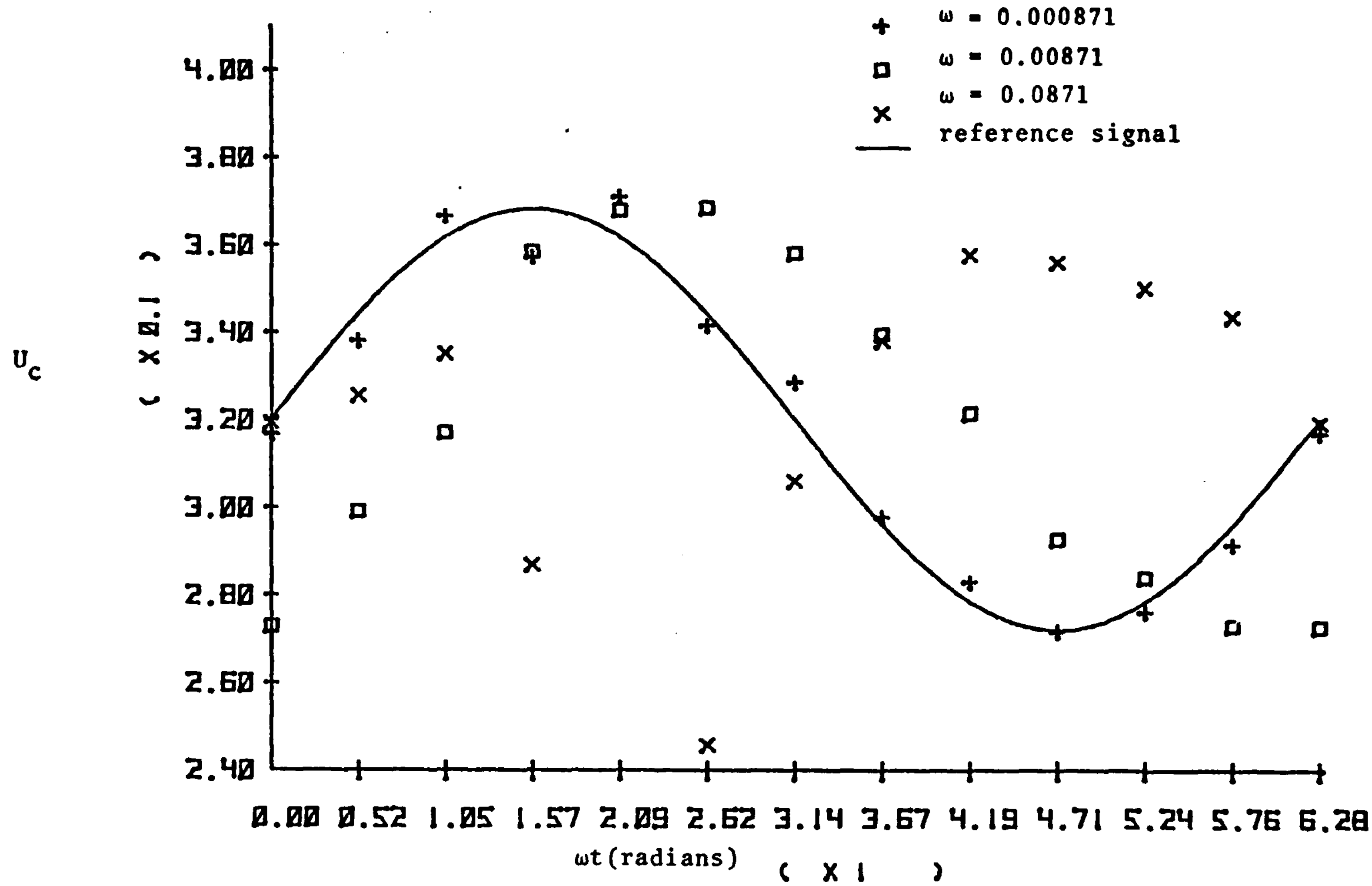

Figure 13(f) Variation of Instantaneous Centre-Line Velocity with time For $\zeta = 60.032$, $\varepsilon = 0.15$

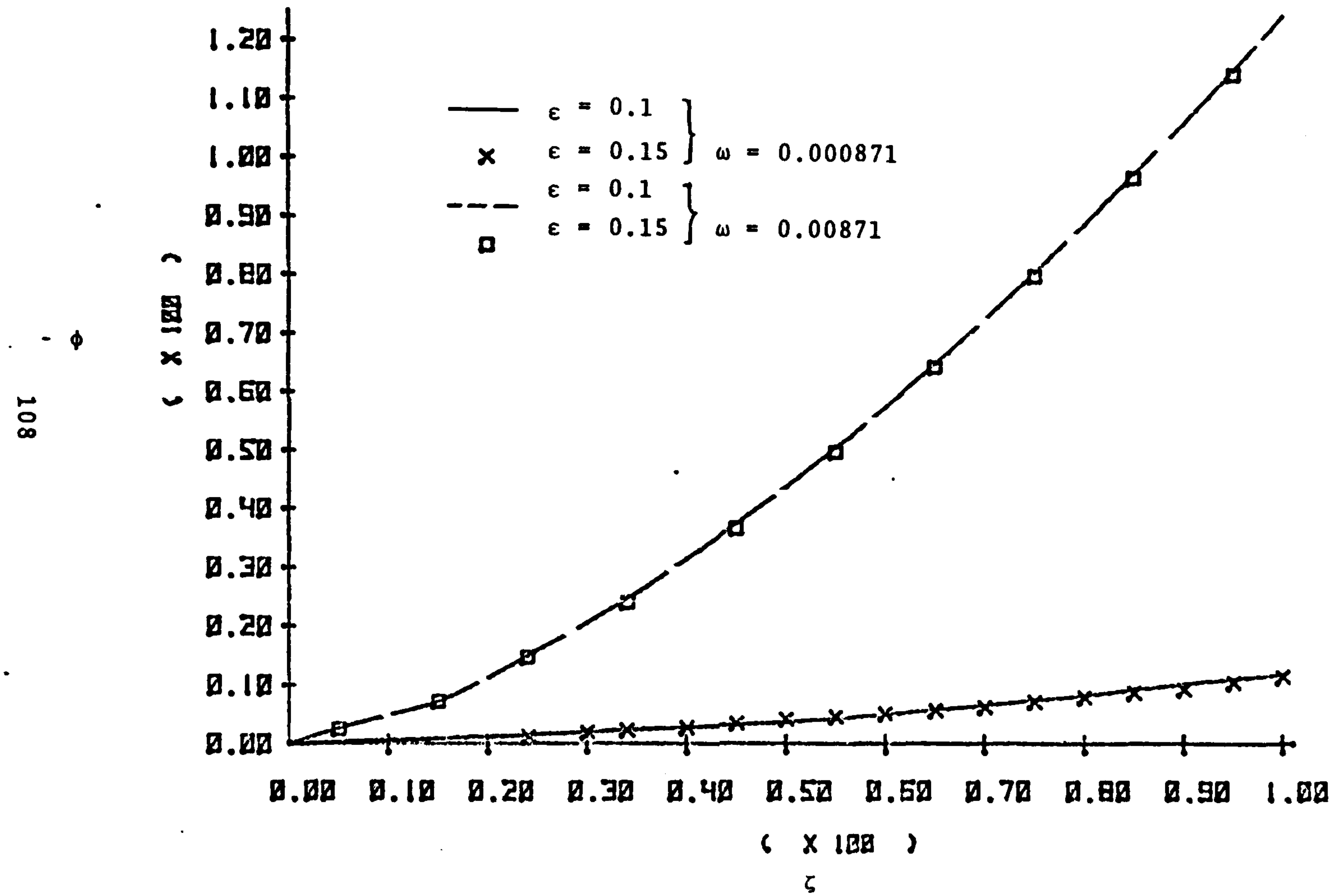

Figure 14(a) Variation of Phase Angle with Streamwise Distance

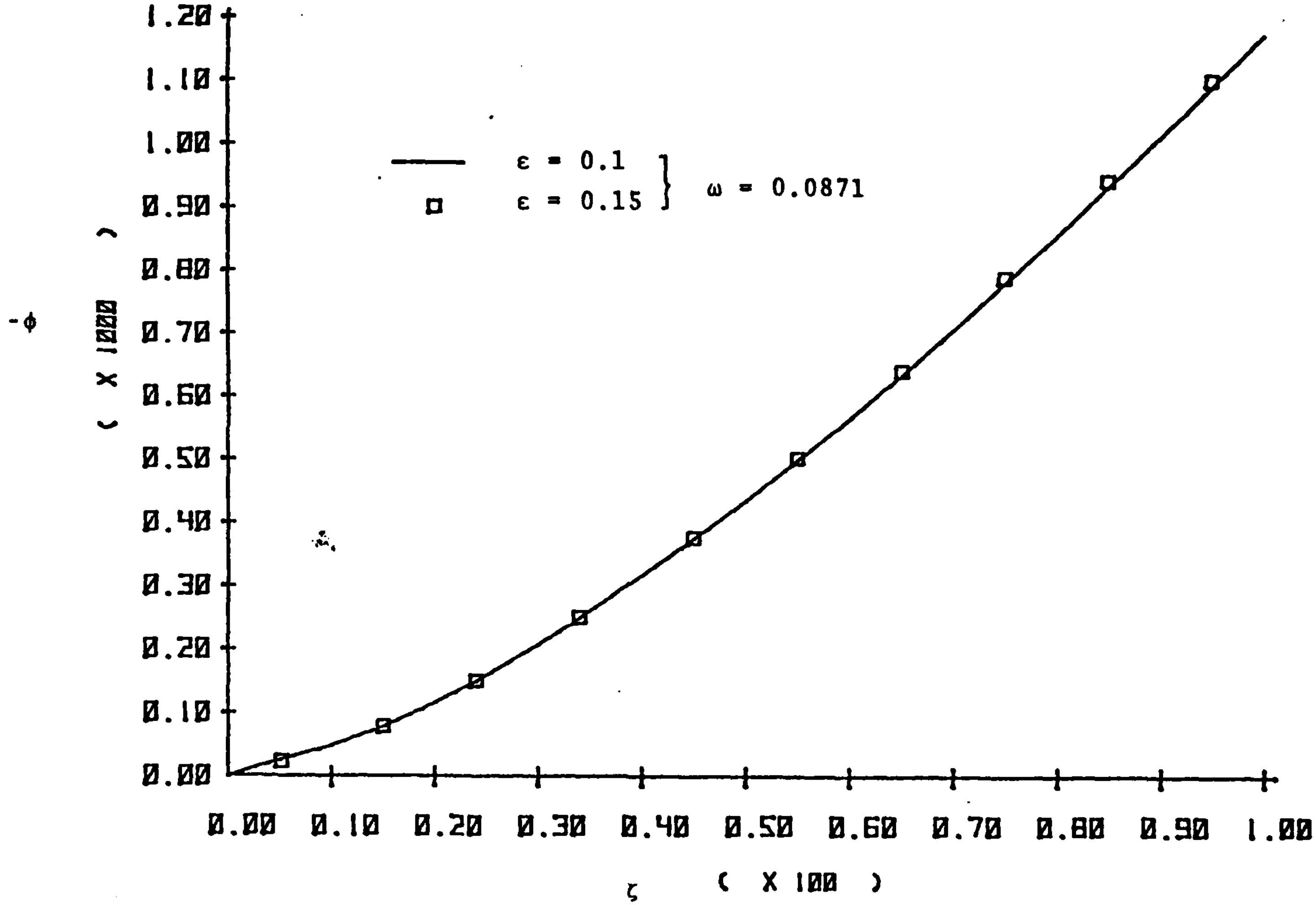

Figure 14(b) Variation of Phase Angle with Streamwise Distance

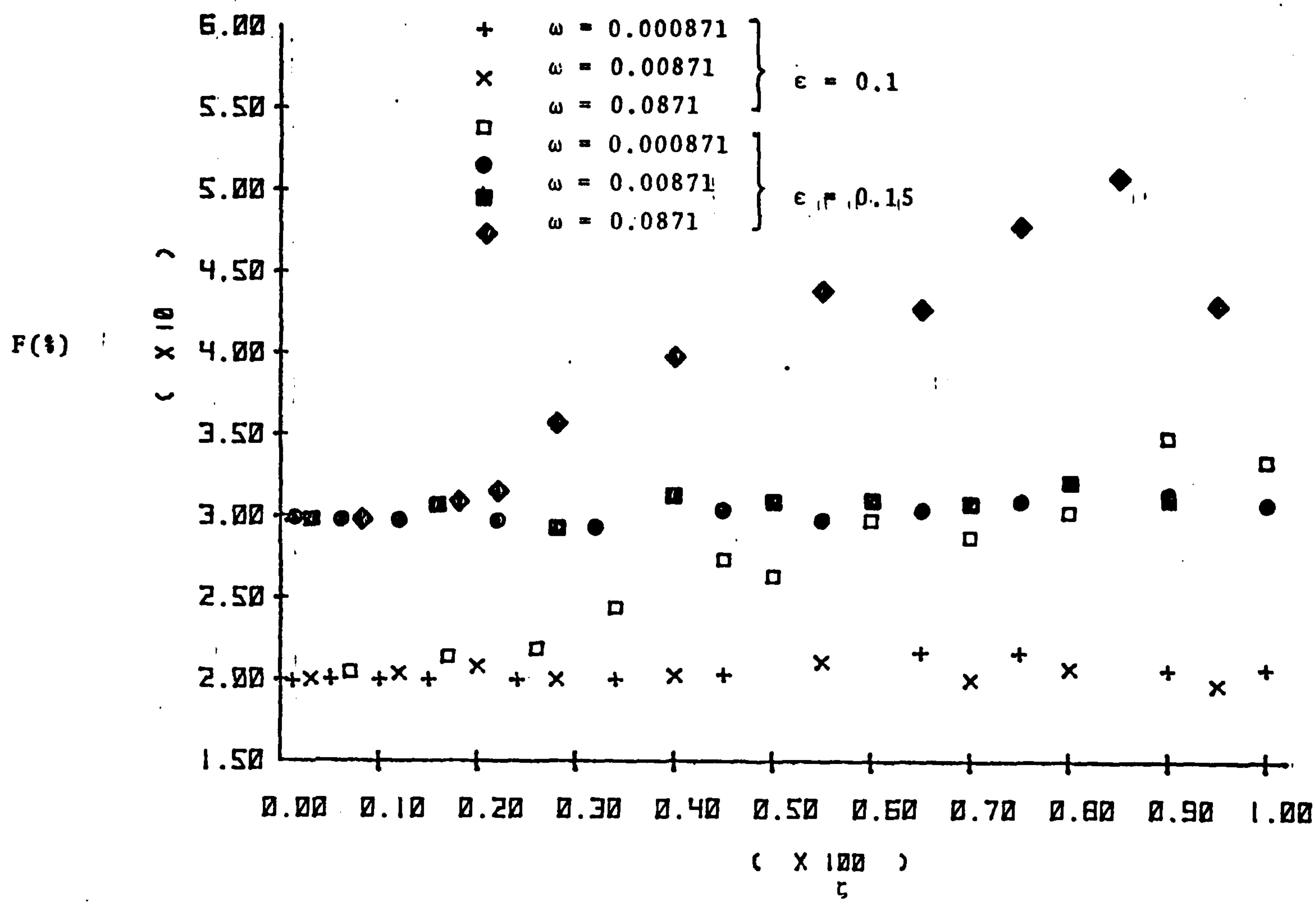

Figure 15 Variation of Peak-to-Peak Oscillation of Centre-Line Velocity with Streamwise Distance

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